Twin Stars:
Neutral Rates and Currency Risk Premia

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Abstract

Under no-arbitrage conditions, the currency risk premium connects the neutral interest rates of two countries. We implement this restriction in a two-country model of exchange rate and foreign bond markets, documenting novel empirical facts on global neutral rates and their role in currency markets: *(i)* global trends in interest rates exhibit strong co-movements across foreign countries, with the U.S. neutral rate in the center of the G9 economies; *(ii)* global neutral rates are characterized by a secular decline, which reverses with the onset of the recent hiking cycle in 2020; *(iii)* cross-country global rate dispersion has gradually narrowed over time; *(iv)* periods of lower average global neutral rates are associated with reduced returns from various carry trade strategies; and *(v)* accounting for interest rate trends across countries helps to refine expected returns in dollars from holding foreign long-term bonds and from currency carry trades. Our findings provide strong evidence that global currency markets interweave foreign and domestic fixed-income markets, highlighting the strong link between these market segments.

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Introduction

The exchange rate between the currencies of two countries can shed light on the long-run trends in interest rates across borders. Our findings trace a middle road between two contrasting perspectives. On one end lies the "exchange rate disconnect" view, which suggests that exchange rates provide little insight into interest rate trends. On the other end, the interest rate parity view posits a strong connection between exchange rates and persistent interest rate differentials. We find that slow-moving variations in the returns to carry trading are intertwined with the differences in long-term interest rate trends across countries.

We analyze and characterize this relationship theoretically using the well-known present-value decomposition (Campbell and Clarida 1987, Dahlquist and Pénasse 2022). Consider $\bar{i}_t$ and $\bar{i}^*_t$, the stochastic trend for the interest rates in the domestic and foreign country, respectively.[1] We show that the present-value decomposition of the exchange rate exists only if the difference $\bar{i}^*_t - \bar{i}_t$ exactly mirrors the stochastic trend $\overline{xcr}_t$ in the returns from a carry trading strategy between these two countries. This is a long-horizon analog to the one-period uncovered interest rate parity. If $\overline{xcr}_{t+1} = 0$, which emerges when investors are risk-neutral and capital flows are unrestricted, then this result implies that there should be no difference between the trends in interest rates between the two countries. If investors are risk averse, then there can be a difference between trends in interest rates but it must be matched with a trend in the currency risk premium.

These results can be challenging to illustrate because the quantities involved are difficult to estimate. To further motivate our analysis, we consider a simple implication that does not rely on further identifying assumptions. We compare the sample means of the interest rate differentials relative to the U.S. with the sample means of excess returns from carry trading across all of the G9 currencies. We find that the average spreads closely line up with the average carry returns, and we find no statistical evidence against the null hypothesis.[2] This unconditional finding is consistent with the results from Hassan and Mano (2018) and may help rationalizing why a static portfolio sorted on the initial level of interest produces most of the average returns from the carry strategy in the cross-section of currencies, while the dynamic portfolio that is rebalanced over time produces small

[1] In the following, the superscript * designates a foreign country variable and the superscript $^-$ designates a stochastic trend, which is defined as $\bar{i}_t = \lim_{n\to\infty} E[i_{t+n}]$.

[2] The G9 currencies are the Australian dollar, the Canadian dollar, the Swiss franc, the euro, the British pound, the Japanese yen, the Norwegian krone, the New Zealand dollar, and the Swedish krona

average returns.

Our analytical and empirical results motivate a joint analysis of trends in interest rates and currency risk premiums. We formalize the risk-return trade-offs underlying this close relationship within a two-country, no-arbitrage framework that integrates exchange rate and bond markets, building on the approach of Chernov and Creal (2023). Our contribution extends this framework to incorporate a transitory and a stochastic trend component in interest rate spreads. Using a no-arbitrage argument, we show that these components of interest rate spreads also determine the transitory and trend component of the exchange rate dynamics, respectively. To complete this framework, we add a flexible term structure model with transitory and trend components in the level of US interest rates, in the spirit of Bauer and Rudebusch (2020) and Feunou and Fontaine (2023).

Empirically, we contribute to the literature studying long-term trends in interest rates. Our results for the U.S. are in line with existing estimates. The interest rate trend is smoother and more persistent in our model, but the pattern shows the well-documented secular decline in long-term rates from around 5 percent at the start of our sample to a low just above 2 percent. The pattern also shares a pronounced reversal after the recovery from the COVID-19 pandemic, by as much of 1.5 percent. These variations highlight the enduring impact of structural factors and the response of neutral rates to economic shifts.

We estimate this two-country model separately for the G9 currencies. Compared to the U.S., the estimated trends are lower for Switzerland, Japan, and the Euro area, but higher for the New Zealand, Australia, Norway and Great Britain. The average difference relative to the U.S. lies between -1.5 to -1 percent for the low interest rate countries and ranges between 1 to 2 percent for the high-interest rate countries. These results align with the persistent interest rate differentials underlying carry strategies in foreign exchange markets.

However, we note three distinct stylized features across our estimates for the G9 currencies. First, despite the differences in levels, the estimated interest rate trends across advanced economies closely follow the pattern we observe in the U.S. Second, the U.S. trend interest rate lies in the center of the group. The difference between the U.S. trend and the mean of the G9 trends averages at 0.04 percent over the sample (standard deviation, 0.21). Third, the dispersion of the trend estimates across the G9 currencies has been gradually narrowing over our sample period. These three features disappear if we re-estimate our models while ignoring the restrictions on the exchange rate dynamics, which underscores the critical role of these restrictions when estimating interest rate trends. There is also a clear disconnect between the observed exchange rates and the rates implied from combining the stochastic discount factors estimated in one-country models. The scale and variability of the implied spot returns are puzzling, reflecting the excess volatility documented by Backus et al. (2001). By contrast, our two-country models match the observed exchange rate dynamics and provide insights about the interest rate trends in economies where exchange rates play a key role in shaping monetary policy and investment conditions.

We also contribute to the literature on carry trading in currency markets by analyzing how portfolio returns are influenced by the average interest rate trends between foreign countries and the United States. Our findings reveal that the performance of time-series carry, cross-sectional carry, and dollar carry trading strategies increases monotonically with the average long-term interest rate trends. Specifically, during periods when long-term interest rates have been low and closely aligned—such as those observed since the global financial crisis—carry trade strategies have generally generated low and insignificant returns. In contrast, at the beginning of the sample, when the average in long-term interest rates was more pronounced, all three strategies delivered consistently positive returns. These results highlight the critical role of long-term interest rate differentials in driving carry trade profitability even for short investment horizons.

Further, we build and expand on the results of Lustig, Stathopoulos, and Verdelhan (2019). Fixing the investment horizon, they show that the predictability of foreign bond returns in dollars declines with the maturity of the bonds and offset the currency risk premium at the longest maturity. Lustig et al. (2019) also translate this result into a important preference-free condition that no-arbitrage models must satisfy. We show that this condition implies that the two countries share the same interest rate trends. However, we build on their analysis and modify this condition for the case when two countries do not share the rate trends. In that case, the difference between the interest rate trends matches the carry trade risk premia in long-term foreign bonds. Consistent with this prediction, we find that the trend differences that we estimate exhibit predictive content for foreign bond returns in dollars that increases with the maturity of the bonds. This predictability is significantly larger relative to benchmark predictability using differences between short-term interest rates, or differences between the slopes of the yield curves. Therefore, accounting for the difference between trends in interest rates can help understand the properties of the carry trade risk premiums.

Literature Review

The secular decline in U.S. bond yields has attracted a long literature that investigates the role of lower real rates or lower inflation rates behind this decline, starting with (Kozicki and Tinsley, 2001; Dewachter and Lyrio, 2006; Rudebusch and Wu, 2008; Bekaert, Cho, and Moreno, 2010) or that analyzes the implications for the behavior of bond risk premium (Cieslak and Povala, 2015; Bauer and Rudebusch, 2020). The empirical literature has since produced estimates of secular declines across advanced economies (Holston, Laubach, and Williams, 2017; Negro, Giannone, Giannoni, and Tambalotti, 2019). We formalize how the exchange rate dynamics connect the some of secular changes in bonds markets across countries.

The affine specification that we use share many similarities with a rich empirical literatures on international bond markets (de los Rios, 2009; Kaminska, Meldrum, and Smith, 2013; Jotikasthira, Le, and Lundblad, 2015; Bekaert and Ermolov, 2023). We also borrow from the substantial literature that infers SDFs from bond returns on domestic and foreign bonds and emphasize a puzzle that a ratio of these SDFs imply an exchange rate that is too volatile (e.g., Backus, Foresi, and Telmer 2001; Ahn 2004; Brennan and Xia 2006; Sarno, Schneider, and Wagner 2012; Chernov and Creal 2023). We build on these two strands of literature to study the theoretical linkages between permanent changes that are common to international bond yields and exchange rates and recover estimate of these secular changes that are consistent with the absence of arbitrage.

Section 1 uses the present-value decomposition to highlight the issues and introduces initial model-free evidence to motivate our work. Section 2 analyzes secular declines across international bond yields in a no-arbitrage framework and the implications for exchange rates. Section 3 closes the specification of the model, introduces identification restrictions and reports the key empirical results.

1 Motivation

1.1 Data and Notations

Data. We obtain the bilateral exchange rates against the U.S. dollar at the end of each month between December 1994 and January 2024 for the G9 currencies: the Australian dollar, the Canadian dollar, the Swiss franc, the euro, the British pound, the Japanese yen, the Norwegian krone, the New Zealand dollar, and the Swedish krona. This sample includes the most liquid currency sport markets and together covers close to 70 percent of the daily trading volume denominated in US dollar (BIS, 2022).

We obtain zero coupon yields for the United States and these G9 currencies, where we use German bonds for the euro. Specifically, we sample bond yields with quarterly maturities of 3, 6, and 9 and 12 months; yearly maturities between 1 and 10 years, and maturities of 15, 20, and 30 years. This breakdown of the yield curves is in part dictated by data availability and offers a detailed comparison of yield curves across countries.

Bond yields and exchange rates. We denote S_t the nominal spot exchange rate of the U.S. dollar per unit of foreign currency and Δs_t the depreciation of the U.S. dollar relative to the foreign currency, in log. We denote $P_t^{(n)}$ and $P_t^{(*n)}$ the prices of domestic and foreign nominal discount bonds, from the point of view of an U.S. investor, paying one unit of domestic and foreign currency at maturity. The continuously compounded yields to maturity are given by $ny_t^{(n)} = -\log(P_t^{(n)})$ and $y_t^{(*n)} = -\log(P_t^{(*n)})$, and we define the short rates $i_t = y_t^{(1)}$ and $i_t^* = y_t^{(*1)}$.

Excess returns. The one-period excess currency returns xcr_{t+1}, given by:

$$xcr_{t+1} = \log\left[\frac{P_t^{(1)}}{P_t^{(*1)}}\frac{S_{t+1}}{S_t}\right] = i_t^* - i_t + \Delta s_{t+1}, \tag{1}$$

is the log excess return of a strategy in which investors borrow at the domestic rate i_t, invest at the foreign rate i_t^*, and convert the proceeds back to U.S. dollars after one period. The domestic one-period excess bond returns $xbr_{t+1}^{(n)}$, given by:

$$xbr_{t+1}^{(n)} = \log\left[\frac{P_{t+1}^{(n-1)}}{P_t^{(n)}}\right] - i_t, \tag{2}$$

is the log excess returns of a domestic strategy in which an U.S. investor borrows for one month at the rate i_t and invests for one period in a nominal bond with maturity n. The foreign one-period excess bond returns $xbr_{t+1}^{(*n)}$ is defined with the same strategy but borrowing at the foreign rate and investing in the foreign bond. Finally, the foreign excess bond returns in dollar, given by:

$$xbr_{t+1}^{\$(n)} = xbr_{t+1}^{(*n)} + xcr_{t+1}. \tag{3}$$

is the log excess return of a strategy in which investors borrow at the domestic rate i_t, invest in a foreign nominal bond with maturity n, and convert the proceeds back to U.S.

dollar after one period. We use the notation xcr_{t+1} and xbr_{t+1} instead of the common and generic form rx_{t+1} to distinguish currency and bond returns.

Trend and Cycle Decompositions. For any random variable z_t such that Δz_t is covariance stationary, the Beveridge-Nelson decomposition is given by:

$$z_t = \bar{z}_t + \tilde{z}_t, \tag{4}$$

where $\bar{z}_t = \lim_{h\to\infty} E_t[z_{t+h}]$ is well defined and $\tilde{z}_t$ is the covariance stationary component with mean zero $E[\tilde{z}_t] = 0$.[3] The component $\bar{z}_t$ is labelled the permanent component, or the shifting endpoint following Kozicki and Tinsley (2001), and $\tilde{z}_t$ is labelled the cyclical or transitory component. Note that we reserve the superscript * notation for foreign variables, following the international finance convention, and the superscript $^-$ for the permanent component in Beveridge-Nelson decompositions.

1.2 Present Value Decomposition

In the following, we assume that each of Δs_t, Δi_t, Δi_t^* and Δxcr_t is covariance stationary. Iterating Equation (1) forward implies the following present-value decomposition:

$$\begin{aligned} s_t &= E_t[s_{t+h}] + \sum_{j=0}^{h-1} E_t\left[\Delta_c i_{t+j}\right] - \sum_{j=1}^{h} E_t\left[xcr_{t+j}\right] \\ &= E_t[s_{t+h}] + h\left(\Delta_c \bar{i}_t - \overline{xcr}_t\right) + \sum_{j=0}^{h-1} E_t\left[\Delta_c \tilde{i}_{t+j}\right] - \sum_{j=1}^{h} E_t\left[\widetilde{xcr}_{t+j}\right], \end{aligned}$$

where we use the Beveridge-Nelson decompositions for i_{t+j}, i_{t+j}^* and xcr_{t+j}, and where $\Delta_c i_t = i_t^* - i_t$ denotes the cross-country difference. This leads to the following motivating result.

Proposition 1 *If Δs_t is covariance stationary, then the exchange rate present-value decomposition in the limit as $h \to \infty$, is given by:*

$$s_t = \lim_{h\to\infty} E_t[s_{t+h}] + \sum_{j=0}^{\infty} E_t\left[\Delta_c \tilde{i}_{t+j}\right] - \sum_{j=1}^{\infty} E_t\left[\widetilde{xcr}_{t+j}\right], \tag{5}$$

and this decomposition only exists if

$$\overline{xcr}_t = \Delta_c \bar{i}_t, \tag{6}$$

[3] If a variable z_t has a constant unconditional mean $E[z] = \bar{z}$, then we have $\lim_{h\to\infty} E_t[z_{t+h}] = \bar{z}$ and $\bar{z}_t = \bar{z}\ \forall t$.

which is equivalent to

$$E_t[\widetilde{xcr}_t] = \Delta_c \tilde{i}_t + E_t[\Delta \tilde{s}_{t+1}]. \tag{7}$$

Note that this charaterizes the Beveridge-Nelson decomposition of the exchange rate $s_t = \bar{s}_t + \tilde{s}_t$, where:

$$\tilde{s_t} = \sum_{j=0}^{\infty} E_t \left[\Delta_c \tilde{i}_{t+j} \right] - \sum_{j=1}^{\infty} E_t \left[\widetilde{xcr}_{t+j} \right], \tag{8}$$

and, therefore, an empirical model of the cyclical interest rate differential $\Delta_c \tilde{i}_t$ and excess currency returns $\widetilde{xcr}_t$ maps into a complete specification of the exchange rate s_t.

If Equation (6) does not hold, then investors must expect that the exchange rate will diverge. To see the intuition, consider the simpler case where excess currency returns are stationary (i.e., $\overline{xcr}_t = 0$). Then, the exchange rate is stationary in level (Backus et al., 2018; Lustig et al., 2019) and it must be that the uncovered interest rate parity hold in the long run $\bar{i}_t = \bar{i}^*_t$, meaning that investors expect that the domestic and foreign short rates will eventually converge to the same value.

If long-run uncovered interest parity does not hold, $\overline{xcr}_t \neq 0$, then the exchange rate is not stationary but, from Equation (6), it must still be the case that the endpoint of expected currency returns corresponds to the difference between the domestic and foreign short rates expected by investors in the long run.[4]

Co-integration relationships. The restriction given by Equation (6) involves unobservable shifting endpoints $\bar{i}^*_t$, $\bar{i}_t$ and $\overline{xcr}_t$ that are challenging to measure or estimate. However, we can evaluate the less restrictive but testable implication linking the averages of interest rates and excess currency returns. Denote $\xi_{t+1} = i^*_t - i_t - E_t[xcr_{t+1}]$, then Equation (6) implies

$$E[\xi_{t+1}] = 0. \tag{9}$$

This testable restriction involves the short rates in both countries but we can obtain similar testable restrictions that involved bond yields in both countries. If we add the assumption that the bond risk premiums are covariance stationary, then the expectation hypothesis holds for bond yields at some arbitrary large horizon, and we obtain the

[4]Contrast this result with the one-period UIP condition where the difference between one-period interest rates is given by a combination of a currency risk premium and expected depreciation.

following decomposition for bond yields:

$$y_t^{(n)} = \bar{i}_t + \tilde{y}_t^{(n)} \quad \text{and} \quad y_t^{(*n)} = \bar{i}_t^* + \tilde{y}_t^{(*n)}, \tag{10}$$

Equation (6) implies additional restrictions involving the average yield differentials:

$$E\left[\xi_{t+1}^{(n)}\right] = 0. \tag{11}$$

where $\xi_{t+1}^{(n)} = y_t^{(*n)} - y_t^{(n)} - E[xcr_{t+1}]$.[5]

Preliminary Tests. Figure 1 compares the sample average excess returns for each currency relative to the U.S. against the average interest rate differential for maturities of 3 months as well as 2, 5, and 10 years across Panels (a)-(d). The results show that average excess currency returns exhibit a wide dispersion across countries, as expected, ranging from 3.1 to -2.1 percent (annualized) for the Japanese yen and the New Zealand dollar, respectively. The average interest rate differentials exhibit a similar dispersion across countries for every bond maturities. For 3-month interest rates, the average differential range between -2.58 to 2.14 percent, respectively for the same two countries.[6] We report the results in Figure 1.

[INSERT FIGURE 1 HERE]

The results exhibit the well-known positive relationship between the average interest rate differentials and average excess carry returns across countries. This is the carry premium: currencies with relatively high interest rates also tend to earn higher carry trading returns (Lustig and Verdelhan, 2007; Lustig et al., 2011). However, the restriction that these averages exactly match has not been analyzed in the literature (to the best of our knowledge). In each panel of Figure 1, the observed spreads and returns are scattered around the 45-degree line defined by $E[\xi_{i,t+1}^{(n)}] = 0$ for country pairs for country i relative to the U.S. We can test if the data support this restriction using a regression of $\xi_{i,t+t1}^{(n)}$ on a constant, pooled across countries. A test that the constant is zero is a test of $E[\xi_{i,t+1}^{(n)}] = 0$.

[5]Hall, Anderson, and Granger (1992) and Engsted and Tanggaard (1994) provide early evidence for the US that bond yields share the same co-integration relationship with the short rate and that the bond risk premiums are stationary. In single-country model of bond yields with shifting endpoints, Bauer and Rudebusch (2020) allow for a more general cointegration structure while Feunou and Fontaine (2023) impose that the bond risk premiums are covariance stationary.

[6]The results are similar using longer maturities are very similar (Panels (b)-(d)), suggesting that the assumption in Equation (10) that expected excess bond returns are stationary may be a useful approximation. We will discuss this possibility in greater details as part of our empirical model.

The standard error of the estimate for this constant accounts for sampling variability in the time-series and in the cross-section.[7] We report the results of this test in Table 1.

[INSERT TABLE 1 HERE]

Panel (A) in Table 1 reports the estimated constant from the pooled regressions and results from a test that $E[\xi_{i,t+1}^{(n)}] = 0$ based on Driscoll-Kraay standard errors that account for potential correlations between the residuals across countries, potential auto-correlations in the error terms of a given country, as well as potential heteroscedasticity. The estimated constant is small not significantly different from zero for every maturity (the p-values range from 0.76 and 0.99).

Panel (B) in Table 1 reports estimates of the currency fixed effects in a pooled regression for the case $n = 10y$. This is a way to ask if one or any of the individual currencies do not satisfy the restriction. The estimates are small and not statistically different from zero in every case. The p-values range between 0.35 and 0.90, for the Swiss franc and the Australian dollar, respectively. An F-test assessing that all partial intercepts are jointly zero returns a p-value of 0.5, which is unsurprising given the result from the pooled regression in Panel (A).

These results are related to the seminal decomposition by Hassan and Mano (2018) of the average returns from carry trading in the cross-section of currencies. Consider the carry strategy with short positions in currencies with a low interest rate and long positions in currencies with a high interest rate (e.g., Lustig, Roussanov, and Verdelhan 2011). Figure 2 reports the annualized average excess return when the portfolios are rebalanced at the end of each month separately for cases where the maturity of the funding instrument varies between three months and ten years. Following Hassan and Mano (2018), we decompose the average returns between a static (grey bars) and a dynamic component (blue bars). The static component corresponds to the average returns from a portfolio constructed at the start of the sample based on the prior historical averages of interest rates. The dynamic component corresponds to the average returns attributed to the monthly rebalancing activity throughout the sample.

[INSERT FIGURE 2 HERE]

[7] It is tempting to use a regression of average returns on average differentials, essentially using least-square to draw a line in Figure 1. However, the standard errors from this regression would ignore the time-series sampling variability, which we expect would be large.

Unsurprisingly the traditional carry trade strategy generates positive returns close to 4 percent, independent of the maturity of the underlying funding instrument. Second, average excess returns slightly decline for yields with maturities longer than 2-years and, in fact, turn weakly significant for the 5- and 10-year based bonds. Third, the vast majority of carry returns stems from the static component that captures the persistent differences illustrated by Figure 1. Lastly, the share of the dynamic component is small and insignificant in every case.

Overall, the results illustrate that the persistent differences between the level of interest rates across countries have considerable implications for the returns from carry trading strategy, both in the time series and in the cross-section. The evidence supports the restriction in Equation (6) closely linking the interest rate shifting endpoints across countries. This motivates us to incorporate this restriction into our empirical strategy in Section 2, which will help identify the properties of excess currency returns and shed light on the differences between interest rate endpoints across countries.

1.3 No-Arbitrage

In this section, we use the absence of arbitrage to establish a connection between the level factors in the yield curves and the variance of the stochastic discount factor across countries, leading to further restrictions that multi-country models bond markets must satisfy.

We follow the asset market view of currency markets. The depreciation rate S_{t+1}/S_t is given by the ratio M^*_{t+1}/M_{t+1}, where M_t and M^*_t are the domestic and foreign Stochastic Discount Factors (SDFs) expressed in units of domestic and foreign currency, respectively. In the spirit of Alvarez and Jermann (2005), each SDF has a decomposition of the following form:

$$M_{t+1} = M^{\mathcal{P}}_{t+1} M^{\mathcal{T}}_{t+1}. \tag{12}$$

Following Lustig, Stathopoulos, and Verdelhan (2019), this decomposition determines important properties of the exchange rate dynamics in relation to properties of the bond yields. However, the decomposition Alvarez and Jermann (2005) covers the case where interest rates and bond yields are stationary. To extent their analysis to the case with potentially non-stationary interest rates, we assume a generic exponential-affine structure for bond prices. First, the short rate is given by $i_t = i_{\zeta,t} + i_{\tau,t}$, where:

$$i_{\zeta,t} = \delta_{0,\zeta} + \delta_\zeta^\top \zeta_t \qquad i_{\tau,t} = \delta_{0,\tau} + \delta_\tau^\top \tau, \tag{13}$$

and

$$\zeta_{t+1} \overset{\mathbb{Q}}{\sim} \mathcal{N}(\mu^{\mathbb{Q}} + \phi^{\mathbb{Q}}\zeta_t, \Omega_\zeta) \qquad \tau_{t+1} \overset{\mathbb{Q}}{\sim} \mathcal{N}(\tau_t, \Omega_\tau), \tag{14}$$

so that the zero-coupon bond prices can be decomposed as follows:

$$P_t^{(n)} = P_{\tau_t}^{(n)} P_{\zeta_t}^{(n)}, \tag{15}$$

where each component is given by:

$$P_{\zeta_t}^{(n)} = \exp(A_{\zeta,n} + B_{\zeta,n}^\top \zeta_t) \qquad P_{\tau_t}^{(n)} = \exp(A_{\tau,n} + n\delta_\tau^\top \tau_t). \tag{16}$$

The distinction between $P_{\zeta_t}^{(n)}$ and $P_{\tau_t}^{(n)}$ is that the loadings $B_{\zeta,n}/n$ of forward rates on ζ_t converge to zero for distant maturities, while the loadings on τ_t is the constant δ_τ and generate a pure level effect across the term structure of interest rates. When τ_t moves up or down, then forward rates exhibit a parallel shift across maturities. This decomposition arises in the broad class of affine dynamic term structure models where the interest rate i_t has a unit root under the risk-neutral measure (see Equation 14). We assume the same structure in the foreign bond markets, but we avoid introducing the parallel notations for now, for parsimony. Proposition 2 characterizes the SDF decomposition when interest rates share a level factors.

Proposition 2 *Given the construction of bond prices in Equations 13-16, then the transitory pricing kernel $\Lambda_{t+1}^{\mathcal{T}}$ is defined as follows:*

$$\Lambda_t^{\mathcal{T}} \equiv \lim_{n\longrightarrow\infty} \frac{\delta^{t+n}}{P_{\zeta_t}^{(n)}},$$

the transitory SDF component $M_{t+1}^{\mathcal{T}}$ is given by:

$$M_{t+1}^{\mathcal{T}} = \frac{\Lambda_{t+1}^{\mathcal{T}}}{\Lambda_t^{\mathcal{T}}} = \exp\left(\ln\delta + \tilde{\lambda}_\zeta^\top \Delta\zeta_{t+1}\right),$$

where $\tilde{\lambda}_\zeta = \delta_\zeta(I - \phi^{\mathbb{Q}})^{-1}$,

$$\ln\delta = \frac{1}{2}Var_t\left[\ln\left(M_{t+1}^{\mathcal{T}}\right)\right] - \tilde{\lambda}_\zeta^\top \mu_\zeta^{\mathbb{Q}} - \delta_{0,\zeta},$$

and the pricing kernel components have drifts given by:

$$E_t^{\mathbb{Q}}[M_{t+1}^{\mathcal{T}}] = \exp\left(-i_{\zeta,t} + Var_t\left[\ln\left(M_{t+1}^{\mathcal{T}}\right)\right]\right)$$
$$E_t[M_{t+1}^{\mathcal{P}}] = \exp(-i_{\tau_t}).$$

Parallel results hold in the foreign bond markets.

Under the asset market view, we have:

$$\Delta s_{t+1} = \Delta\bar{s}_{t+1} + \Delta\tilde{s}_{t+1} = \Delta_c \ln M_{t+1}^{\mathcal{P}} + \Delta_c \ln M_{t+1}^{\mathcal{T}}. \tag{17}$$

Therefore, a no-arbitrage specification of the bond markets following Equations (13)-(16) in two countries also pins down the exchange rate dynamics. In addition, from using Proposition 2, the bond market specifications imply a decomposition of the exchange rate dynamics between its transitory and permanent components.[8] Finally, Proposition 2 implies the following parametric restrictions connecting the SDFs across countries.

Starting with the cyclical component, since we have that $\Delta\tilde{s}_{t+1} = \Delta_c \ln M_{t+1}^{\mathcal{T}}$ is covariance stationary, then it must be that $\Delta_c \ln \delta = 0$ and, therefore, the following cross-country restriction must hold:

$$\frac{1}{2}\Delta_c Var_t\left[\ln\left(M_{t+1}^{\mathcal{T}}\right)\right] = \Delta_c\left(\tilde{\lambda}_\zeta^\top \mu_\zeta^{\mathbb{Q}} + \delta_{0,\zeta}\right). \tag{18}$$

Equation (18) connects differences in risks captured by the transitory SDFs across countries with differences between the parameters of the transitory components of the short rates. This condition must hold even if the interest rate dynamics does not include a level factor or a stochastic trend. Either way, relaxing this constraint introduces a deterministic drift in the exchange rate.

Turning to the permanent components, the fact that the SDFs are log-normal implies that:

$$E_t\left[\Delta\bar{s}_{t+1}\right] = \Delta_c \ln\left(E_t\left[M_{t+1}^{\mathcal{P}}\right]\right) - \frac{1}{2}\Delta_c Var_t\left[\ln\left(M_{t+1}^{\mathcal{P}}\right)\right]. \tag{19}$$

The permanent component of each SDF is a martingale in the decomposition of Alvarez and Jermann (2005), because interest rates are stationary. In Proposition 2, the permanent component evolves around a stochastic trend $E_t[M_{t+1}^{\mathcal{P}}] = \exp(-i_{\tau_t})$ because the interest

[8] A substantial body of evidence documents that the permanent component of the SDF is highly economically significant (Alvarez and Jermann, 2005; Bakshi and Chabi-Yo, 2012; Christensen, 2017; Qin et al., 2018) and, in addition Sandulescu et al. (2021) show that the cross-country correlations are relatively low due to international market segmentation.

rates are not stationary and, therefore:

$$E_t[\Delta \bar{s}_{t+1}] = 0 \Leftrightarrow \Delta_c i_{\tau,t} = \frac{1}{2}\Delta_c Var_t\left[\ln\left(\bar{M}_{t+1}\right)\right]. \tag{20}$$

These restrictions will hold by construction in our empirical framework below.

2 Empirical Framework

We develop a multi-country no-arbitrage model of bonds and currencies markets. Our approach puts together core building blocks from the rich literatures on the term structure of interest rates and on exchange rates, but we also leverage the close connection between properties of the exchange rate and of the interest rates endpoints across countries.

2.1 Asset Pricing in Bonds and Currency Markets

Domestic factors dynamics. The vector x_t denotes the N_x state variables that are relevant in the domestic bond market, where Δx_t is covariance stationary, with a Beveridge-Nelson decomposition given by $x_t = \bar{x}_t + \tilde{x}_t$. We define the conditional expectation $\bar{\mathcal{E}}_{x,t} = E_t[\bar{x}_{t+1}]$ and $\tilde{\mathcal{E}}_{x,t} = E_t[\tilde{x}_{t+1}]$ separately, with:

$$\mathcal{E}_{x,t} = E_t[x_{t+1}] = \bar{\mathcal{E}}_{x,t} + \tilde{\mathcal{E}}_{x,t}, \tag{21}$$

and $\varepsilon_{x,t+1} = x_{t+1} - \mathcal{E}_{x,t}$ are gaussian innovations with covariance matrix Ω.

First, the yield factors share the same shifting endpoint, given by:

$$\begin{aligned} \bar{\mathcal{E}}_{x,t} &= \gamma \bar{\tau}_t \\ \bar{\tau}_{t+1} &= \bar{\tau}_t + \bar{\sigma}^\top \varepsilon_{x,t+1}, \end{aligned} \tag{22}$$

where $\bar{\tau}_t$ is a scalar, γ is an $N \times 1$ vector and, consistent with the definition of a shifting endpoint, we have $E_t[\Delta \bar{\mathcal{E}}_{t+1}] = 0$. This specification follows Cieslak and Povala (2015), Bauer and Rudebusch (2020) and Feunou and Fontaine (2023), where bond yields share the same co-integration relationship.

Second, the dynamics for the cyclical component $\tilde{\mathcal{E}}_t$ are given by:

$$\tilde{\mathcal{E}}_{x,t+1} = \mu + \phi \tilde{\mathcal{E}}_{x,t} + \psi \varepsilon_{x,t+1}. \tag{23}$$

This specification implies that the forecast of the bond yields are not spanned by the current term structure of yields. This follows Feunou and Fontaine (2018), who show that such cyclical dynamics closely match the bond risk premium measured from predictive regressions and improve forecasts of bond returns. Overall, the conditional means are available recursively from observing the time series x_t given the initial values $\bar{\mathcal{E}}_{x,0}$ and $\tilde{\mathcal{E}}_{x,0}$.

Domestic bond market. The domestic short rate is given by:

$$i_t = \delta_0 + \delta^\top x_t. \tag{24}$$

We split the state vector x_t as follows:

$$x_t = \begin{bmatrix} \tau_t \\ \zeta_t \end{bmatrix}, \tag{25}$$

and we construct the domestic state dynamics under the risk-neutral measure $\mathbb{Q}$ such that τ_t acts as a pure "level" factor across bond yields. Specifically, we assume the following risk-neutral dynamics:

$$\begin{aligned} \tau_{t+1} &= \tau_t + \varepsilon^{\mathbb{Q}}_{\tau,t+1} \\ \zeta_{t+1} &= \mu^{\mathbb{Q}}_{\zeta} + \phi^{\mathbb{Q}}_{\zeta}\zeta_t + \varepsilon^{\mathbb{Q}}_{\zeta,t+1}, \end{aligned} \tag{26}$$

where by construction $\varepsilon^{\mathbb{Q}}_{x,t+1} = [\varepsilon^{\mathbb{Q}}_{\tau,t+1}\ \varepsilon^{\mathbb{Q}}_{\zeta,t+1}]$. Using standard results, the zero-coupon bond prices are given by:

$$P^{(m)}_t = \exp\left(-A_m - B_m^\top x_t\right), \tag{27}$$

which can also be decomposed as in Equations (15)-(16). The zero-coupon yields are given by $y^{(m)}_t = a_m + b_m^\top x_t$, where the coefficients (A_m, B_m) and (a_m, b_m) obey first-difference equations. This specification belongs to the class of affine dynamic term structure models, which provide a high degree of flexibility to capture the cross-section and time-series properties of domestic and foreign bond yields. See e.g., Duffie and Kan (1996); Dai and Singleton (2000); Joslin, Singleton, and Zhu (2011).

Foreign factors dynamics. The vector x^*_t denotes the N_{x^*} state variables that are relevant for the spreads between domestic and foreign yields, where Δx^*_t is covariance stationary and, therefore, $x^*_t = \bar{x}^*_t + \tilde{x}^*_t$. In addition, the vector z_t stacks together the excess currency returns with the yields spreads factors, as follows:

$$z_t \equiv \begin{bmatrix} xcr_t \\ x^*_t \end{bmatrix},$$

because the dynamics of the exchange rate and yield spreads are connected by the absence of arbitrage, and they have correlated innovations, given by:

$$\varepsilon_{z,t+1} = z_{t+1} - \mathcal{E}_{z,t}, \tag{28}$$

with covariance matrix Ω^*. Stacking the excess currency returns with the bond spreads factors means that the exchange rate dynamics can have innovations that are uncorrelated with the yield factors innovations (i.e., not spanned by bond yields) following Chernov and Creal (2023). Including this source of variations that are not spanned by bond returns is essential to capture the time-series properties of exchange rates and resolves both the FX volatility and forward premium anomalies. Here, we do not take a stand on the interpretation of these unspanned variations in our reduced-form specification: it could reflect a limitation due to building estimates of the pricing kernel as projections on bond price variations, or it could reflect the limits of investors' set of opportunities due to incomplete markets.

Like in the domestic market, the permanent component of the spreads $\bar{x}_t^*$ share a common shifting endpoint, such that $\bar{\mathcal{E}}_{x^*,t} = \gamma^* \bar{\tau}_t^*$ where γ^* is a vector and $\bar{\tau}_t^*$ is a scalar with dynamics given by:

$$\bar{\tau}_{t+1}^* = \bar{\tau}_t^* + \bar{\sigma}^{*\top} \varepsilon_{z,t+1}. \tag{29}$$

Again, like in the domestic market, the cyclical components of the spreads $\tilde{x}_{t+1}^*$ have dynamics given by:

$$\tilde{\mathcal{E}}_{x^*,t+1} = \mu^* + \phi^* \tilde{\mathcal{E}}_{x^*,t} + \psi^* \varepsilon_{z,t+1}, \tag{30}$$

where, similar to the domestic factors, the conditional means are available recursively.

Foreign bond market. The spread between the foreign and domestic short-term interest rates is given by:

$$\Delta_c i_t = \delta_0^* + \delta^{*\top} x_t^*. \tag{31}$$

The risk-neutral distribution of the domestic bond factors x_t is the same in each country, consistent with our identification of x_t^* with yield spreads. We split the state vector x_t^* as follows:

$$x_t^* = \begin{bmatrix} \tau_t^* \\ \zeta_t^* \end{bmatrix}, \tag{32}$$

and we construct the domestic state dynamics under the risk-neutral measure $\mathbb{Q}^*$ such that τ_t^* acts as a pure "level" factor across yields spreads. Specifically, we assume the following risk-neutral dynamics:

$$\begin{aligned}\tau^*_{t+1} =& \tau^*_t + \varepsilon^{\mathbb{Q}^*}_{\tau^*,t+1} \\ \zeta^*_{t+1} =& \mu^{\mathbb{Q}*}_{\zeta^*} + \phi^{\mathbb{Q}*}_{\zeta^*}\zeta^*_t + \varepsilon^{\mathbb{Q}^*}_{\zeta^*,t+1},\end{aligned} \tag{33}$$

where by construction $\varepsilon^{\mathbb{Q}}_{x^*,t+1} = [\varepsilon^{\mathbb{Q}}_{\tau^*,t+1}\ \varepsilon^{\mathbb{Q}}_{\zeta^*,t+1}]$. We obtain that the pricing equation for foreign zero-coupon bond prices, expressed in units of foreign currencies, is given by:

$$P^{*(m)}_t = \exp\left(-A_m - B_m^\top x_t\right)\exp\left(-A^*_m - B^{*\top}_m x^*_t\right) = P^{(m)}_t \exp\left(-A^*_m - B^{*\top}_m x^*_t\right), \tag{34}$$

where (A^*_m, B^*_m) obey first-difference equations. This construction of the foreign short rate as a spread to the domestic short rate is a normalization where yield spreads are affine in the foreign state vector: $D_c y^{(m)}_t \propto x^*_t$ and, of course, $i^*_t = i_t + \Delta_c i_t$. This is a convenient choice because the domestic and foreign bond markets interact with the exchange rate equation via these yields spreads.

Exchange rate. The empirical specification of bond prices across countries pins down the dynamics of $\widetilde{xcr}_{t+1}$ and $\overline{xcr}_{t+1}$. First, given the definition of the short rate in each country and the assumption that the spread variables share the same random walk, then the long-horizon interest rate parity condition $\overline{xcr}_{t+1} = \Delta_c \bar{i}_{t+1}$ in Proposition 1 implies that:

$$\overline{xcr}_{t+1} = \delta^{*\top}\gamma^*\bar{\tau}^*_{t+1}. \tag{35}$$

Second, Proposition 2 and the fact that the exchange rate component $\Delta\tilde{s}_{t+1} = \Delta_c \ln M^{\mathcal{T}}_{t+1}$ is stationary implies the following cyclical exchange rate component:

$$\widetilde{xcr}_{t+1} = \Delta_c\widetilde{i}_t + \tilde{\lambda}^\top_{\zeta^*}\Delta\zeta^*_{t+1}, \tag{36}$$

where $\tilde{\lambda}_{\zeta^*} = \delta_{\zeta^*}(I - \phi^{\mathbb{Q}*}_{\zeta^*})^{-1}$.[9] Putting these results together, we can show that the exchange rate dynamics in our specification are given by:

$$\Delta s_{t+1} = \tilde{\lambda}^\top_{\zeta^*}\Delta\zeta^*_{t+1} + \psi^\top_s \varepsilon_{z,t+1}, \tag{37}$$

where $\psi^\top_s = \delta^\top\gamma\bar{\sigma}^{*\top}$. Consistent with the role of the interest rate differential and yield spreads in the present-value decomposition, the spread factors play a central role in Equation (37). It should also be clear what we use the stacked innovations $\varepsilon_{z,t+1}$ to capture the exchange rate variations that are unspanned in bond returns, as in Chernov and Creal

[9]Using Proposition2, we have the transitory components dynamics $\ln M^{\mathcal{T}}_{t+1} = \ln\delta + \tilde{\lambda}^\top_\zeta\Delta\zeta_{t+1}$ and $\ln M^{*\mathcal{T}}_{t+1} = \ln\delta^* + \tilde{\lambda}^\top_\zeta\Delta\zeta_{t+1} + \tilde{\lambda}^\top_{\zeta^*}\Delta\zeta^*_{t+1}$ and $\Delta\tilde{s}_{t+1}$ is stationary if and only if $\Delta_c \ln\delta = 0$.

(2023).

2.2 Estimation

U.S. bonds. The cross-section of J domestic zero-coupon yields $y_t^{(m)} = -\log(P_t^{(m)})/m$ stacked in the vector Y_t can be expressed in terms of its N leading principal components $\mathcal{P}_t$, given by:

$$\mathcal{P}_t = WY_t = Wa + Wbx_t = C + Dx_t, \tag{38}$$

where W is the $N \times J$ matrix of loadings, and where a and b are stacking the yields coefficients across maturities, as follows:

$$a = \begin{bmatrix} a_{m_1} \\ \vdots \\ a_{m_J} \end{bmatrix} \qquad b = \begin{bmatrix} b_{m_1}^\top \\ \vdots \\ b_{m_J}^\top \end{bmatrix},$$

Foreign bonds. The cross-section of J^* spreads $\Delta_c y_t^{(m)}$ between foreign and domestic yields stacked in the vector $\Delta_c Y_t$ can be expressed in terms its N^* leading principal components $\mathcal{P}_t^*$, given by:

$$\mathcal{P}_t^* = W_c \Delta_c Y_t = W_c a^* + W_c b^{*\top} x_t^* = C^* + D^* x_t^*, \tag{39}$$

where W_c is the $N^* \times J^*$ matrix of coefficients. If the principal components are measured without errors, we obtain the yields and spreads factors $x_t = D^{-1}(\mathcal{P}_t - C)$ and $x_t^* = D^{*-1}(\mathcal{P}_t^* - C^*)$ given the yields coefficients (a, b) and (a^*, b^*). We also re-express the pricing equations in terms of the principal components, as follows:

$$y_t^{(m)} = a_{\mathbb{P},m} + b_{\mathbb{P},m}^\top \mathcal{P}_t \qquad y_t^{(*m)} = y_t^{(m)} + a_{\mathbb{P},m}^* + b_{\mathbb{P},m}^{*\top} \mathcal{P}_t^* \tag{40}$$

where $a_{\mathbb{P},m} = a_m - b_m^\top D^{-1} C$ and $b_{\mathbb{P},m}^\top = b_m^\top D^{-1}$. This no-arbitrage representation of bond yields parallels the canonical work in Joslin, Singleton, and Zhu (2011) but in a multi-country setting. In the spirit of Joslin et al. (2011), the necessary restrictions to achieve econometric identification of the parameters are implemented under the risk-neutral measure. Appendix AII provides details about the estimation and identification.

3 Results

We estimate the two-country model separately for each of the G9 currencies relative to the U.S. Our result for $\bar{i}_t$ in the U.S. are similar to existing estimates. However, our results for $\bar{i}_t^*$ across other advanced countries differ from the previous studies and, therefore, provide a unified perspective on secular changes in interest rate and the role of exchange rates. The flip side of these results is that the spreads in the interest rate trends $\Delta_c \bar{i}_t^*$ from a two-country model offers better estimates of long-horizon currency and bond risk premia. The Internet Appendix provides details about the estimation and additional estimates and statistics from the model.

3.1 U.S. Natural Rate of Interest

For the U.S. bond market, we offer a specification that is close in spirit to existing term structure models. Unsurprisingly, our estimates closely line up with earlier studies. To see this, we report in Figure 3 our estimate of $\bar{i}_t$ for the U.S., together with existing estimates.[10] We find that all the estimates of the nominal stochastic trend $\bar{i}_t$ move closely with each other, along a gradual decline from around 5 percent at the start of our sample to about 2 percent during the outbreak of the Covid-19 pandemic.

[INSERT FIGURE 3 HERE]

Panel A in Table 2 provides summary statistics to underscore these observations. The sample mean of $\bar{i}_t$ is 3.2 percent in our model, which is within the range of sample means in other models (between 3.1 and 3.6 percent). In the last column, we report the sample mean for the difference between our estimate and each alternative measure. The average differences are small and range between 0.1 and 0.3 percent across the nominal estimates.[11] The compensation for expected inflation produces a level shift between the estimates of the nominal and real trends.

The estimates also exhibit a similar degree of volatility, around 1 percent annually or less. If anything, our estimate tends to exhibit a smoother and more persistent path. The

[10] We include nominal estimates from Christensen and Rudebusch (2012), Christensen and Rudebusch (2016), and Kim and Wright (2005) as well as real estimates from Holston et al. (2017), Hördahl and Tristani (2014) and Lubik and Matthes (2015). We thank Jens Christensen for providing us real rate estimates. Nominal rate estimates are obtained from https://www.bis.org/publ/qtrpdf/r_qt2403b.htm.

[11] Note that the average difference is not the same as the difference between the reported averages because the sample periods vary across estimates. The average differences are calculated for the overlapping sample period, during which both estimates are available.

first-order autocorrelation is 0.98 in our model but it ranges between 0.93 and 0.96 for the other estimates. The LM2015 estimate stands out with a lower autocorrelation.[12]

[INSERT TABLES 2 HERE]

Panel B reports the correlations between trend estimates $\bar{i}_t$ for US interest rates. The results show that our estimate exhibits high correlations that range between 0.82 and 0.87 with nominal estimates and between 0.88 and 0.93 with real estimates. Overall, Figure 3 and Table 2 suggest that our U.S. estimates for the nominal interest rate trend $\bar{i}_t$ that we estimate is consistent with existing results.

3.2 Global Natural Rates of Interest

In Figure 4, we report the estimated stochastic trends estimated across the set of G9 currencies. We obtain the estimate by estimating the two-country model from Section 2 separately for each currency. For simplicity, we fix the estimated interest rate dynamics for the U.S. and we estimate the two-country model to fit the dynamics of the exchange rate and foreign interest rate spreads.

In Panel (a), we compare the U.S. estimate with the average estimate SoE^{Avg} across the currencies in our sample. The cross-country average is remarkably close to the U.S. estimate. Both series exhibit the same gradual decline, as well as the same reversals around 2005 and more recently since 2022. This small difference does not arise because the individual estimates are the same. Panel (a) also reports the range of estimates across countries (grey-shaded area). There is wide dispersion between the estimates throughout the sample, around two percent above or below the U.S. estimates.

This small difference between the U.S. and cross-country average is an empirical stylized fact. This fact emerges because we use the exchange rates to estimate our two-country model. The models are estimated separately across currencies and the strong commonality that we recover across the trend estimates arises from some common features across the exchange rate dynamics.

[INSERT FIGURE 4 HERE]

Another feature of the results in Panel (a) of Figure 4 is that the cross-country dispersion in the shaded area is gradually declining over time. This dispersion peaked close to 4.1

[12] The LM2015 estimated series exhibit the lowest correlations with all other estimates.

percent in the mid-90s and declined to around to 2.8% in 2020. The dispersion has increased since the pandemic, closer to three percent, but remains lower than in the early part of the sample. This pattern is correlated with the level of the interest rate trends. Therefore, one possibility is that the lower bound under nominal interest rate is mechanically compressing this range when the trends are declining and approaching zero. However, another possibility indicated from Equation (20), is that the narrowing reflects smaller differences between of the quantity of systematic risks across countries.

We report the individual trend estimates $\bar{i}_t^*$ separately in Panel (b) of Figure 4. There are two clear features in this picture. First, countries with high interest rate (AUD, NZD, GBP) exhibit stochastic trends that remain above that of the U.S. consistently through the sample, while countries with low interest rates (CHF, EUR, JPY) exhibit trends below that of the U.S. Panel (A) of Table 3 reports summary statistics for the interest rate differentials $\Delta_c \bar{i}_t$ relative to the U.S. Consistent with the literature on carry trades, the typical carry trade funding currencies CHF and JPY exhibits a large negative average differential of -1.65% and -1.62%, respectively, while, at the other end of the spectrum, the AUD and NZD exhibit average differential of 1.18% and 2.15%, respectively. Note that the persistent spread from the model across these currency pairs resembles the empirical pattern presented in Figure 1.

[INSERT TABLE 3 HERE]

The second feature that is apparent in Figure 4 is that every foreign trend follows a path that is very similar to the trend that we observed for the U.S. The figure clearly points towards a decline in long-term rates across all countries and they all share a similar reversal after the COVID recover. Panel (B) of Table 3 reports the correlations across these trends. The lowest correlation with the U.S. trend is 0.94 (GBP and JPY) and the lowest correlation across all currency pairs 0.85.

The timing of some of the features can differ across currencies. For economies with historically low interest rates, such as CHF (green) and JPY (light blue), a decline in the estimates is apparent in the mid-90s already, while estimates for Germany and most other European countries display noticeable declines only after the great financial crisis. This declining trend is partly offset towards the end of the sample across all of our estimates. We see a reversal of around one percent in most cases. For instance, the estimates show a reversal from around 0 to 1 percent for the CHF, from around 4.5 to 5.5 percent for the NZD.

3.3 The Role of the Exchange Rate Restrictions

We can evaluate the role of the exchange rate restrictions on the estimated trend. In Figure 5, we compare the estimates from a conventional one-country model (dotted red line) and from our two-country model (solid black line), separately for each currency. The differences between estimates across the two models are visible and especially stark for low interest rate currencies. For the CHF, JPY, and EUR currencies, we find that the secular decline in long-term rates are noticeably more pronounced for the two-country model. By contrast, the one-country model appears to miss the significant decline around 2008 financial crisis. The differences are less stark for the high interest rate currencies but nonetheless can still reach up two percent. In sum, accounting for the dynamics of the bilateral exchange rate with the USD produces differences between estimates of the interest rate trends that are economically meaningful.

Excess Volatility. The one- and two-country models also imply different exchange rate dynamics. Our two-country model matches the exchange rate by construction. We can also recover an implied exchange rate when combining two one-country models estimated separately for the US and one of the advanced economies in our sample. The implied exchange rate is implied from the estimated SDF in each country, as follows:

$$\Delta s_{t+1} = \frac{M^*_{t+1}}{M_{t+1}}. \tag{41}$$

Figure 6 illustrates in scatter plots the monthly depreciation rate Δs_{t+1} against the depreciation implied by the combination of one-country models. We also draw the best linear fit (red-dashed line) as a visual gauge of the correlation. The results show that implied values Δs_{t+1} from one-country models have low correlations with realized values across all countries. The regression line is flat in every case. Second, the scale and variability of the sport returns are much larger in the model relative to the realize values, which is a replication of the excess volatility puzzle revealed by Backus et al. (2001). Third, the spot returns implied by combining one-country models exhibit a positive bias. This stark difference makes clear that the exchange rate dynamics offer potentially important information to discipline our two-country model and impose clear restriction on the joint model of the bond markets across countries, on the long-horizon estimates that we recover.

[INSERT FIGURE 6 HERE]

Carry trades and $\bar{i}_t$ differential. We check the close relationship between the differential $\Delta_c \bar{i}_t$ and the risk premium from trading the carry across currencies. For this purpose, we follow Chernov, Dahlquist, and Lochstoer (2023) and implement three well-known trading strategies. First, we implement "time-series" carry (TS-Carry). Following Burnside, Eichenbaum, Kleshchelski, and Rebelo (2010), we form a portfolio each month with long and short positions in currencies with a positive forward discount with respect to the U.S. dollar, respectively. This creates a time-varying exposure to the U.S. dollar, since the number of foreign currencies with long positions changes over time. Second, we implement the dollar-carry strategy (DOL-Carry). Following Lustig, Roussanov, and Verdelhan (2014), we form a portfolio each month with only long positions against the US dollar when the average forward discount is positive, and with only short positions when the average is negative. Third, we implement the cross-sectional carry strategy (CS-Carry), in which the weight of a currency in the portfolio is given by the difference between its forward discount and the average across currencies, following Hassan and Mano (2018).[13]

Consistent with existing results, we find that every strategy produces significant returns in our sample (TS-Carry: 3.2 percent, DOL-Carry: 3.3 percent, CS-Carry: 4.0 percent). However, the pattern changes across sub-samples. We report in Figure 7, the average annualized excess returns for each of the three carry trading strategies in a separate panel. For each strategy, we separately report the average returns in three sub-samples of observations when $\bar{i}_t^{*,avg}$ is either "Low", "Medium" or "High", as shown on the x-axis. Looking back to Figure 4, the "High" sub-sample corresponds to observations mostly early in the sample when the cross-country average of $\bar{i}_t^*$ estimates is higher than the estimate for the U.S. According to the results in Section 1, long positions in the foreign positions should offer higher, positive expected returns. Similarly, the "Low" sub-sample corresponds to observations mostly late in the sample when the cross-country average is lower than in the U.S. In this case, theory predicts that short positions in foreign currencies should earn lower expected returns.

The results reveal a clear monotone pattern across all three strategies. Specifically, the average returns is close to zero and insignificant in sub-sample when $\Delta_c \bar{i}_t^{avg}$ is low but the average is larger when $\Delta_c \bar{i}_t^{avg}$ is high and positive. Furthermore, the 10 percent confidence intervals indicate that the average is significantly different from zero for the TS-carry and the Dollar-carry when $\Delta_c \bar{i}_t^{avg}$ is high but not in other sub-samples. However, the pattern is

[13] In contrast to the original high-minus-low portfolio sorting approach by Lustig et al. (2011) the linear-weighting ensures that the strategy includes all currency pairs in the carry portfolio.

not statistically significant for the CS-carry strategy that does not keep balanced profile of long and short positions against the U.S. dollar. Overall, this result underscores that the difference between long-horizon forward rates is an important signal of the profitability of short-term carry trading strategies.

[INSERT FIGURE 7 HERE]

In Figure 8, we report the cumulative returns from each carry trading strategy separately, but in each case scaled with an initial \$1 investment at the beginning of the sample period. We also highlight with different colors the periods when $\Delta_c \bar{i}_t^{avg}$ is either "low", "medium" or "high". A few key observations emerge. First, the three strategies exhibit a clear commonality, especially early in the sample. During this period, from 1995 to around the 2008 financial crisis, the strategies are visually nearly indistinguishable and earn a steep average returns. Figure 8 also shows that this period largely corresponds to the sub-sample when the cross-country average differential $\Delta_c \bar{i}_t^{avg}$ is high.

Second, this period is followed by more varied performances during the medium $\Delta_c \bar{i}_t$ period, with returns becoming less uniform across strategies. Notably, while all strategies performed similarly until around 2005, the TS-Carry strategy subsequently exhibited relatively lower returns compared to the CS-Carry and DOL-Carry strategies.

Third, during periods of low $\Delta_c \bar{i}_t$—particularly since 2014—all three strategies show weak performance, as indicated by declining or flat return indices. This reflects the diminished profitability of carry trades when long-term interest rate differentials are lower. Although there is some recovery in the indices following the COVID-19 pandemic, the overall performance of the strategies remains noticeably lower than in the earlier years of the sample.

[INSERT FIGURE 8 HERE]

In summary, both figures underscore the pivotal role of $\Delta_c \bar{i}_t$ in driving the performance of carry trading strategies over time. Periods of high $\Delta_c \bar{i}_t$ are consistently associated with common and positive returns, as the favorable interest rate differentials amplify the profitability of borrowing in low-interest-rate currencies and investing in high-interest-rate currencies. These periods allow all three strategies to capitalize on the strong divergence in long-term interest rates, leading to significant compounding of returns.

Long bond predictability. The no-arbitrage framework yields clear predictions that should help compare different estimates of the spread between two interest rate trends. From Equation (20), this spread should provide the best estimate of the wedge between the risk premium on long horizon bonds across these two currencies. Therefore, we use the spread between the level factors to revisit the time-series predictability regressions from Lustig, Stathopoulos, and Verdelhan (2019) to evaluate the predictability of domestic excess bond returns, foreign bond excess return in dollars, and foreign exchange excess returns.

Specifically, Lustig et al. (2019) first analyzes the predictability of the bond returns differentials in dollar $xbr_{t+1}^{\$(n)} - xbr_{t+1}^{(n)}$. We repeat their analysis using the difference between the levels of interest rate across countries. Panel A of Table 4 presents the regression results across countries, fixing the bond maturity to 30 years. Each column corresponds to one currency, while the last panel reports the common coefficient estimate in a panel regression with currency fixed effects. The results reveal that the level differential is a robust and statistically significant predictor of dollar bond excess returns differential. The coefficient estimates range from 4.78 for the euro to 31.01 for the New Zealand dollar, with t-statistics indicating statistical significance in every case. In the panel regressions, we observe a coefficient estimate of 13.13, significant at the one percent level, underscoring the predictive power of the level factor across a broad set of currencies.

[INSERT TABLE 4 HERE]

In Panel B we report results using the level factor differential to forecast the excess currency returns xcr_{t+1}. The results are weak and inconsistent. The panel regression yields an estimate of 0.16, which is statistically indistinguishable from zero. The individual coefficients are not statistically significant with two exceptions but with opposite signs. There is no evidence in our sample that the level factor differential predictive power for currency excess returns.

In Panel C, we analyze the predictability of the bond local currency excess returns $xbr_{t+1}^{*(n)} - xbr_{t+1}^{(n)}$. We find that the level factor differential is a significant predictor independent of the currency of denomination. Overall, the evidence shows that the returns difference in dollar between investing in domestic and foreign bonds is mostly due to the difference between interest rate trends and not to the predictable exchange rate risk premium.

We repeat this analysis across bond maturities from 6 months to 30 years, but focusing on results from the panel regression with currency fixed effects. We report the results visually in Figure 9. Panel (a) reports the slope coefficients along with their corresponding 10 percent confidence intervals in predictive regression for the excess bond returns differential in dollar. We find that the magnitude of the coefficients increases monotonically, starting from approximately 1.5 at the 6-month maturity and rising to 13 for 30-year maturities. Using the 10-percent level, the coefficient reaches statistical significance starting with maturities of five years or more. This pattern suggests that the level factor's predictive power strengthens as bond maturity increases, reinforcing its relevance for longer-term bond return dynamics.

[INSERT FIGURE 9 HERE]

In Panel (b), we report the coefficient in predictive regressions for the excess currency returns. The coefficients are positive and decreases monotonically as the maturity increases, from 0.6 to 0.16. However, the standard errors remain broad across all specifications, reflecting high variability in the estimates for currency returns. In Panel (c), we report the results coefficients in predictive regressions for the excess bond returns differential in local currency. The results confirm that the excess returns differential in dollar is essentially all due to differences between the bond risk premium and not to the dynamics of the exchange risk premium.

The predictability evidence contrast with the results from Lustig, Stathopoulos, and Verdelhan (2019), where they find that the returns to currency carry trades decrease as the maturity of the foreign bonds increases but the local currency term premia, increases with the maturity of the bonds and offset the currency risk premia. They provided ample evidence using the difference between short-term interest rates or the difference between the steepness of the term structure between currencies. In our case, we are guided by the prediction from the no-arbitrage framework that it is the difference between the level in long-horizon interest rates that shares the close connection with the bond returns dollar differential in dollars.

4 Conclusion

In this paper, we leverage the relationship between currency risk premia and long-term trends in fixed income markets to estimate neutral rates. Specifically, we apply a bilateral bond market model across major G9 advanced economies and the United States, providing long-term estimates that account explicitly for the interconnectedness of key financial markets. Our findings offer new insights into the global dynamics of neutral rates.

First, we benchmark our model with U.S. estimates drawn from prior studies, revealing similar time-series dynamics. Our estimates exhibit a secular decline over the past two decades, followed by a reversion after the COVID-19 pandemic. Over the entire sample period, the U.S. natural rate is 3.22%, and even though it is highly persistent, it fluctuates between 2% and 5%.

Second, we observe a parallel decline in natural rates across major economies, comparable to that seen in the United States. While the average natural rate across G9 economies aligns with the U.S. level, significant divergence exists among smaller open economies. Countries typically associated with funding currencies, such as the Switzerland and Japan, exhibit lower natural rates than those linked with investment currencies, such as the Australia or New Zealand. Consistent with our model, this cross-country divergence in estimates lines up with average interest rate differentials that are typically used to form currency carry trade portfolios in foreign exchange markets.

Third, our analysis underscores the importance of considering exchange rate dynamics when estimating neutral rates for small open economies. In particular, for low-interest-rate countries, neutral rate estimates diverge substantially when traditional single-country models are used. Further, we show how these conventional models are unable to match exchange rate dynamics. This suggests that currency dynamics play a significant role in shaping neutral rate estimates.

To conclude, our study highlights the critical need for a globally interconnected approach in assessing neutral rates, especially in the context of small open economies. The findings suggest that overlooking cross-market influences, particularly in currency dynamics, may lead to incomplete or inaccurate assessments of long-term neutral rates.

References

Ahn, D.-H. (2004). Common factors and local factors: Implications for term structures and exchange rates. *Journal of Financial and Quantitative Analysis 39*(1), 69–102.

Alvarez, F. and U. J. Jermann (2005). Using asset prices to measure the persistence of the marginal utility of wealth. *Econometrica 73*(6), 1977–2016.

Backus, D., N. Boyarchenko, and M. Chernov (2018). Term structures of asset prices and returns. *Journal of Financial Economics 129*(1), 1–23.

Backus, D. K., S. Foresi, and C. I. Telmer (2001). Affine term structure models and the forward premium anomaly. *The Journal of Finance 56*(1), 279–304.

Bakshi, G. and F. Chabi-Yo (2012). Variance bounds on the permanent and transitory components of stochastic discount factors. *Journal of Financial Economics 105*(1), 191–208.

Bauer, M. and G. D. Rudebusch (2020). Interest rates under falling stars. *American Economic Review 110*(5), 1316–1354.

Bekaert, G., S. Cho, and A. Moreno (2010). New keynesian macroeconomics and the term structure. *Journal of Money, Credit and Banking 42*(1), 33–62.

Bekaert, G. and A. Ermolov (2023). International yield comovements. *Journal of Financial and Quantitative Analysis 58*(1), 250–288.

BIS (2022). Triennial Central Bank survey–foreign exchange turnover in april 2022. Technical report, Bank for International Settlements.

Brennan, M. J. and Y. Xia (2006). International Capital Markets and Foreign Exchange Risk. *The Review of Financial Studies 19*(3), 753–795.

Burnside, C., M. Eichenbaum, I. Kleshchelski, and S. Rebelo (2010, 12). Do peso problems explain the returns to the carry trade? *The Review of Financial Studies 24*(3), 853–891.

Campbell, J. Y. and R. H. Clarida (1987). The dollar and real interest rates. *Carnegie-Rochester Conference Series on Public Policy 27*, 103–139.

Chernov, M. and D. Creal (2023). International yield curves and currency puzzles. *The Journal of Finance 78*(1), 209–245.

Chernov, M., M. Dahlquist, and L. Lochstoer (2023). Pricing currency risks. *The Journal of Finance 78*(2), 693–730.

Christensen, J. H. E. and G. Rudebusch (2016). Modeling yields at the zero lower bound: Are shadow rates the solution? In *Dynamic Factor Models*, Volume 35, pp. 75–125.

Christensen, J. H. E. and G. D. Rudebusch (2012). The response of interest rates to us and uk quantitative easing. *The Economic Journal 122*(564), F385–F414.

Christensen, T. M. (2017). Nonparametric stochastic discount factor decomposition. *Econometrica 85*(5), 1501–1536.

Cieslak, A. and P. Povala (2015). Expected returns in treasury bonds. *The Review of Financial Studies 28*(10), 2859–2901.

Dahlquist, M. and J. Pénasse (2022). The missing risk premium in exchange rates. *Journal of financial economics 143*(2), 697–715.

Dai, Q. and K. J. Singleton (2000). Specification analysis of affine term structure models. *The Journal of Finance 55*, 1943–1978.

de los Rios, A. D. (2009). Can affine term structure models help us predict exchange rates? *Journal of Money, Credit and Banking 41*(4), 755–766.

Dewachter, H. and M. Lyrio (2006). Macro factors and the term structure of interest rates. *Journal of Money, Credit and Banking*, 119–140.

Duffie, D. and R. Kan (1996). A yield-factor model of interest rates. *Mathematical Finance 6*, 379–406.

Engsted, T. and C. Tanggaard (1994). Cointegration and the US term structure. *Journal of Banking and Finance 18*(1), 167–181.

Feunou, B. and J.-S. Fontaine (2018). Bond risk premia and gaussian term structure models. *Management Science 64*(3), 1413–1439.

Feunou, B. and J.-S. Fontaine (2023). Secular economic changes and bond yields. *The Review of Economics and Statistics 105*(2), 408–424.

Hall, A. D., H. M. Anderson, and C. W. J. Granger (1992). A cointegration analysis of Treasury bill yields. *The Review of Economics and Statistics 74*(1), 116–126.

Hassan, T. A. and R. C. Mano (2018, 10). Forward and spot exchange rates in a multi-currency world. *The Quarterly Journal of Economics 134*(1), 397–450.

Holston, K., T. Laubach, and J. C. Williams (2017). Measuring the natural rate of interest: International trends and determinants. *Journal of International Economics 108*, S59 – S75. 39th Annual NBER International Seminar on Macroeconomics.

Hördahl, P. and O. Tristani (2014, September). Inflation Risk Premia in the Euro Area and the United States. *International Journal of Central Banking 10*(3), 1–47.

Joslin, S., K. J. Singleton, and H. Zhu (2011). A new perspective on Gaussian dynamic term structure models. *Review of Financial Studies 24*, 926–970.

Jotikasthira, C., A. Le, and C. Lundblad (2015). Why do term structures in different currencies co-move? *Journal of Financial Economics 115*(1), 58–83.

Kaminska, I., A. Meldrum, and J. Smith (2013). A global model of international yield curves: No-arbitrage term structure approach. *International Journal of Finance & Economics 18*(4), 352–374.

Kim, D. H. and J. H. Wright (2005). An arbitrage-free three-factor term structure model and the recent behavior of long-term yields and distant-horizon forward rates. Finance and Economics Discussion Series 2005-33, Board of Governors of the Federal Reserve System (US).

Kozicki, S. and P. A. Tinsley (2001). Shifting endpoints in the term structure of interest rates. *Journal of Monetary Economics 47*(3), 613–652.

Lubik, T. A. and C. Matthes (2015). Calculating the Natural Rate of Interest: A Comparison of Two Alternative Approaches. *Richmond Fed Economic Brief* (Oct).

Lustig, H., N. Roussanov, and A. Verdelhan (2011). Common risk factors in currency markets. *Review of Financial Studies 24*(11), 3731–3777.

Lustig, H., N. Roussanov, and A. Verdelhan (2014). Countercyclical currency risk premia. *Journal of Financial Economics 111*(3), 527–553.

Lustig, H., A. Stathopoulos, and A. Verdelhan (2019, December). The term structure of currency carry trade risk premia. *American Economic Review 109*(12), 4142–77.

Lustig, H. and A. Verdelhan (2007). The cross-section of foreign currency risk premia and consumption growth risk. *American Economic Review 97*, 89–117.

Negro, M. D., D. Giannone, M. P. Giannoni, and A. Tambalotti (2019). Global trends in interest rates. *Journal of International Economics 118*, 248–262.

Qin, L., V. Linetsky, and Y. Nie (2018, 04). Long forward probabilities, recovery, and the term structure of bond risk premiums. *Review of Financial Studies 31*(12), 4863–4883.

Rudebusch, G. D. and T. Wu (2008). A macro-finance model of the term structure, monetary policy and the economy. *The Economic Journal 118*(530), 906–926.

Sandulescu, M., F. Trojani, and A. Vedolin (2021). Model-free international stochastic discount factors. *The Journal of Finance 76*(2), 935–976.

Sarno, L., P. Schneider, and C. Wagner (2012). Properties of foreign exchange risk premiums. *Journal of Financial Economics 105*(2), 279–310.

Figures

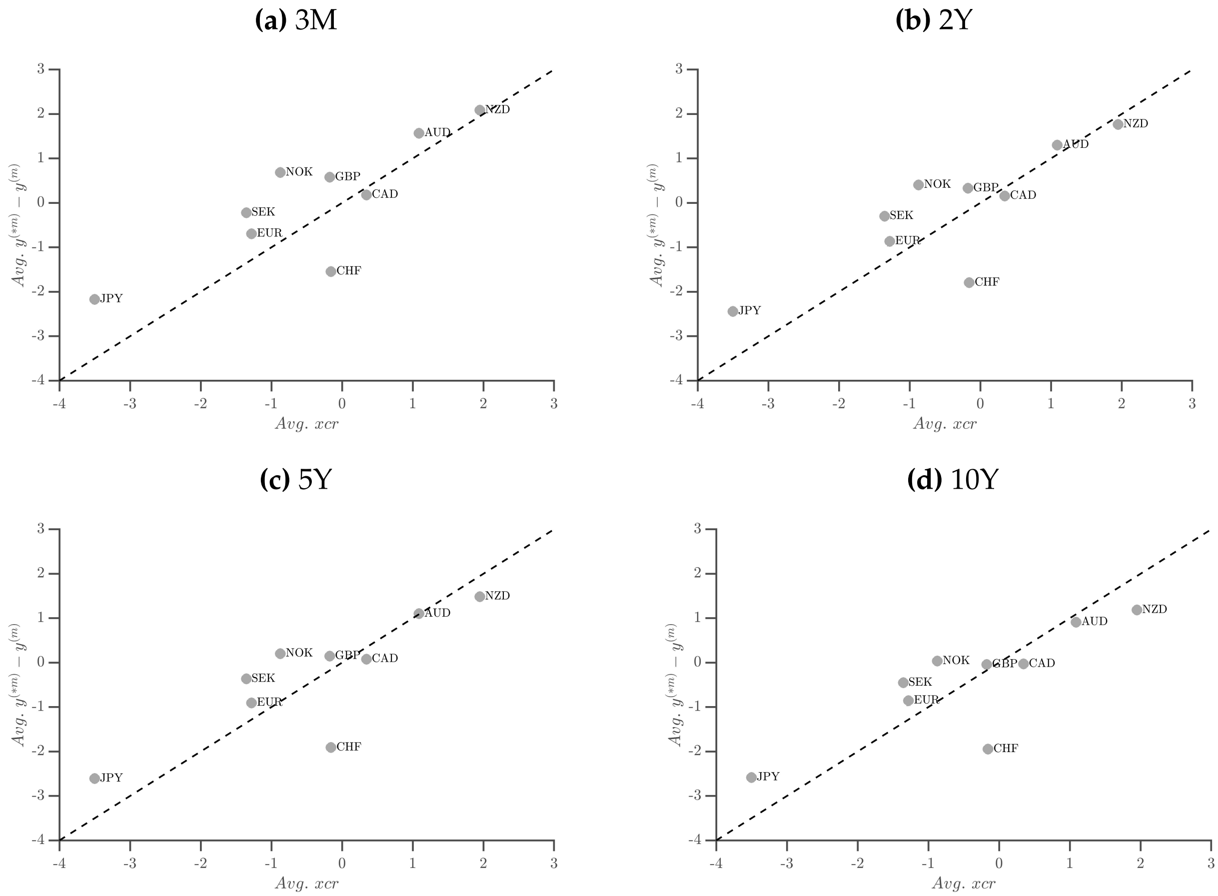

Figure 1: Sample Averages of Yield Differentials and Currency Excess Returns
The figures reports the sample averages of excess currency returns xcr_{t+1} for the G9 currencies on the x-axis and the sample averages of yield differentials $Avg.\ y^{(*n)} - y^{(n)}$) on the y-axis for the maturities $n = 3$ months in Panel (a), $n = 24$ months in Panel (b), $n = 60$ months in Panel (c), and $n = 120$ months in Panel (d). The currencies are the Australian dollar (AUD), the Canadian dollar (CAD), the Swiss franc (CHF), the euro (EUR), the British pound (GBP), the Japanese yen (JPY), the Norwegian krone (NOK), the New Zealand dollar (NZD), and the Swedish krona (SEK). The sample period is January 1995 to January 2024 (349 monthly observations.) All values are expressed in annualized percentage.

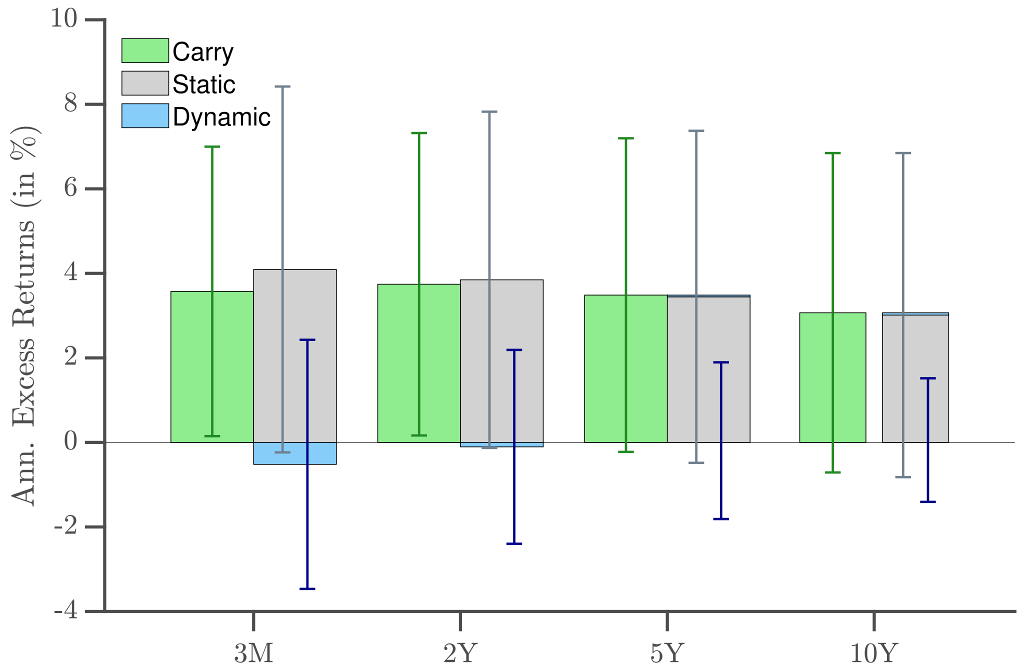

Figure 2: Carry Trade Decomposition
The figure shows annualized excess return to a carry trade strategy (green) for different maturities m - 3-months (3M), 2-years (2Y), 5-years (5Y) and 10-years (10Y). The grey and blue graphs refer to the static and dynamics carry subcomponents, following the decomposition by Hassan and Mano (2018). Error bars refer to 95% confidence intervals. Values are expressed in annualized percentage. The sample period is January 1995 to January 2024.

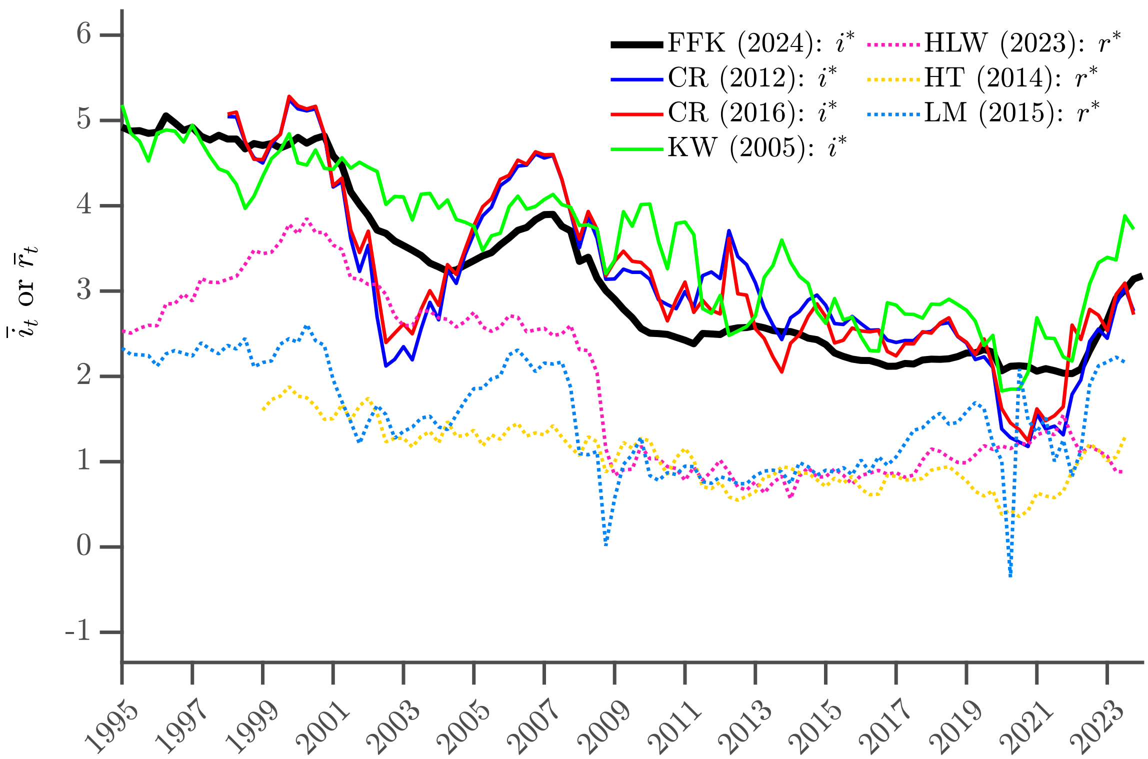

Figure 3: U.S. $\bar{i}_t$ and $\bar{r}_t$ Dynamics
The figure shows the time series dynamics of various natural rate of interest for the United States. The thick black line is the nominal estimate from the model introduced in Section 2. Alternative estimate of the nominal trend are from Christensen and Rudebusch (2012), Christensen and Rudebusch (2016), and Kim and Wright (2005). Estimates of the real trend are from Holston et al. (2017), Hördahl and Tristani (2014) and Lubik and Matthes (2015). Values are expressed in annualized percentage. The sample period is January 1995 to January 2024.

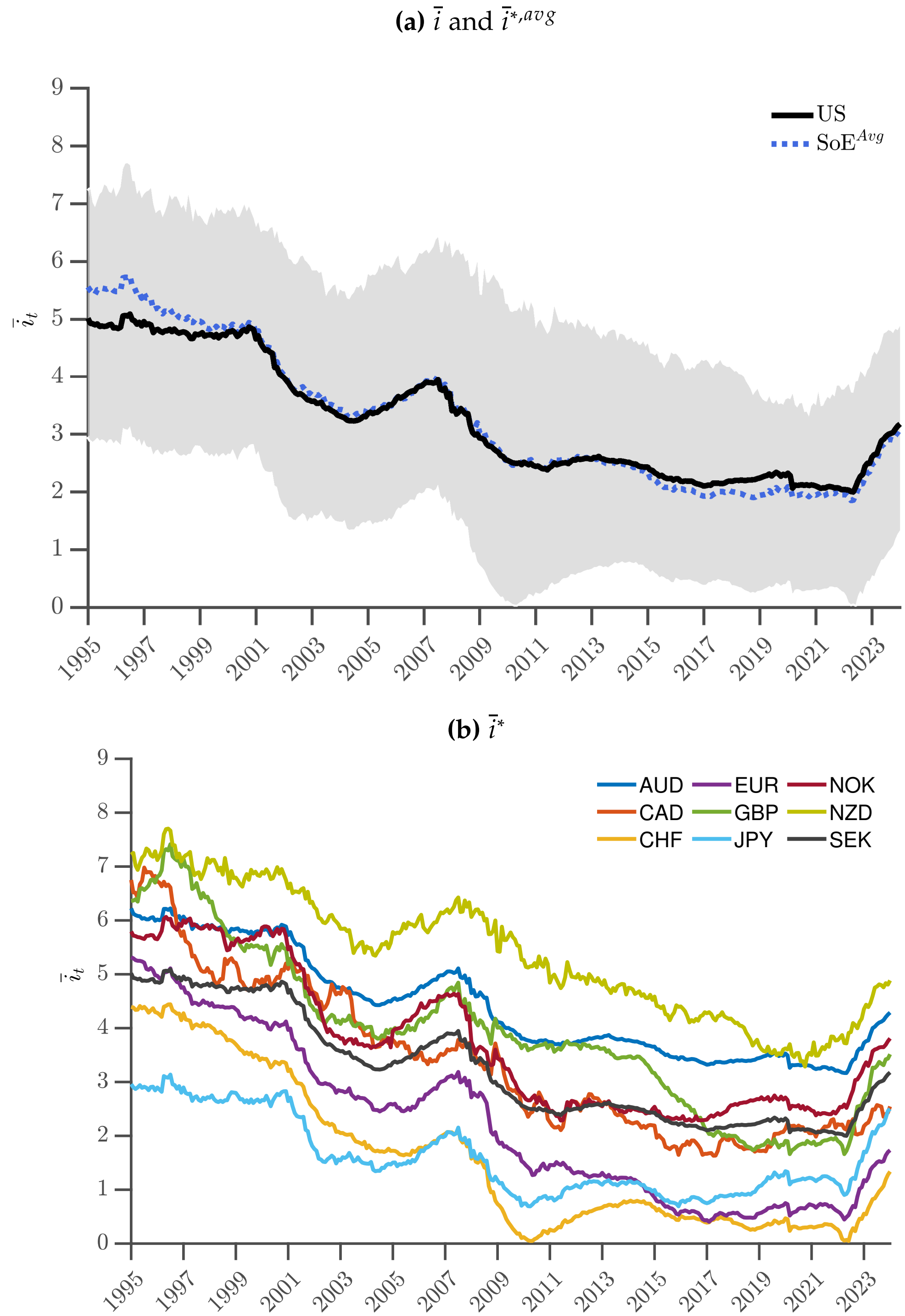

Figure 4: Dynamics of $\bar{i}_t$ across Currencies
The figure shows the global time series dynamics of $\bar{i}_t$. The top panel shows $\bar{i}_t$ for the United States (black line) and the average across a set of major foreign countries, i.e., $\bar{i}_t^{*,avg}$ (dotted blue line). The the grey-shaded area indicates the range of $\bar{i}_t^{*}$ estimates for the set of foreign countries open economies: Australia, Canada, Euro Area, Japan, Norway, New Zealand, Sweden, Switzerland, and United Kingdom. The bottom panel shows $\bar{i}_t^{*}$ separately for all the foreign countries. Values are expressed in annualized percentage. The sample period is January 1995 to January 2024.

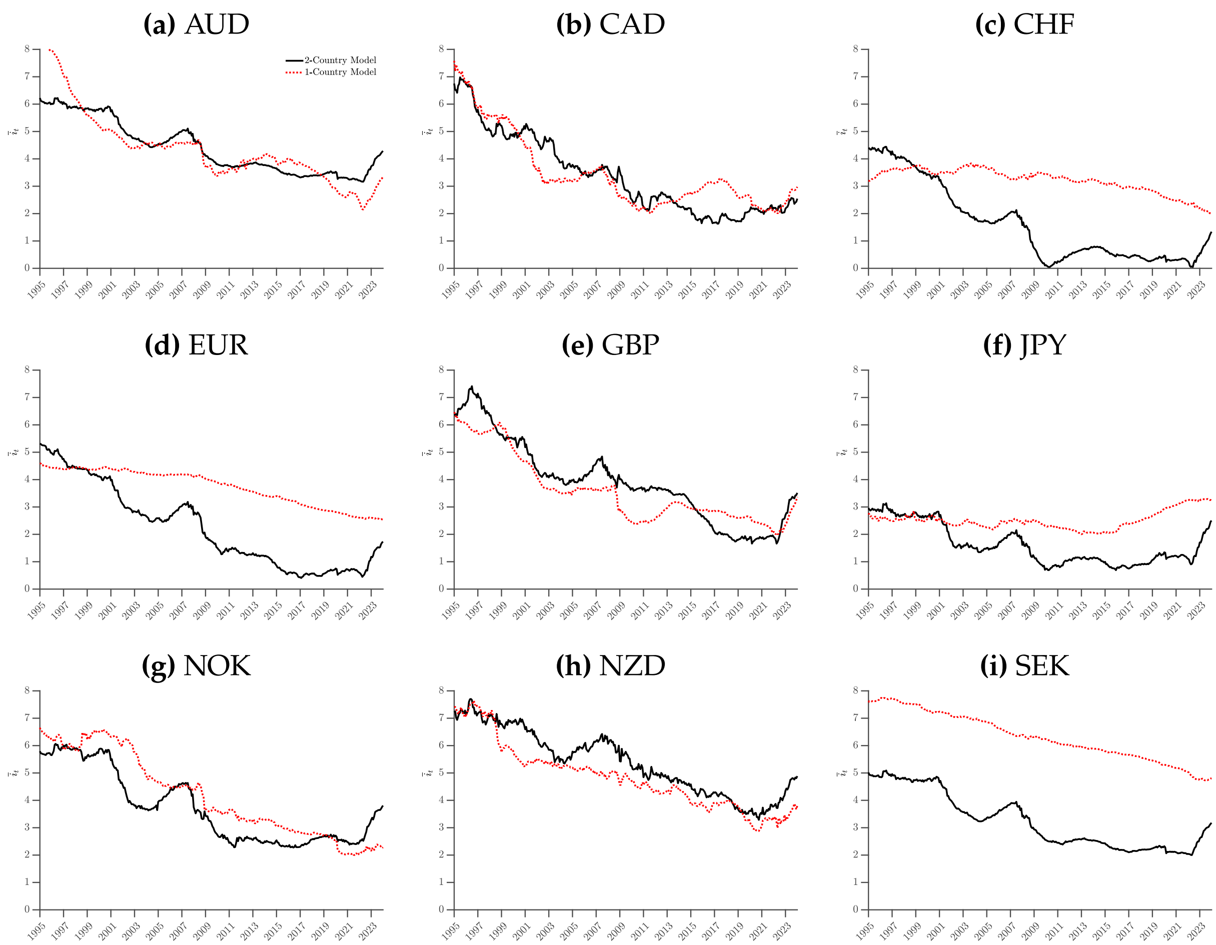

Figure 5: Interest Rates Trend Estimates across G9 Currencies–One-Country and Two-Country Models
The figure shows the estimated trends $\bar{i}_t^*$ across the G9 currencies from the one-and two-country models. Values are expressed in annualized percentage. The sample period is January 1995 to January 2024.

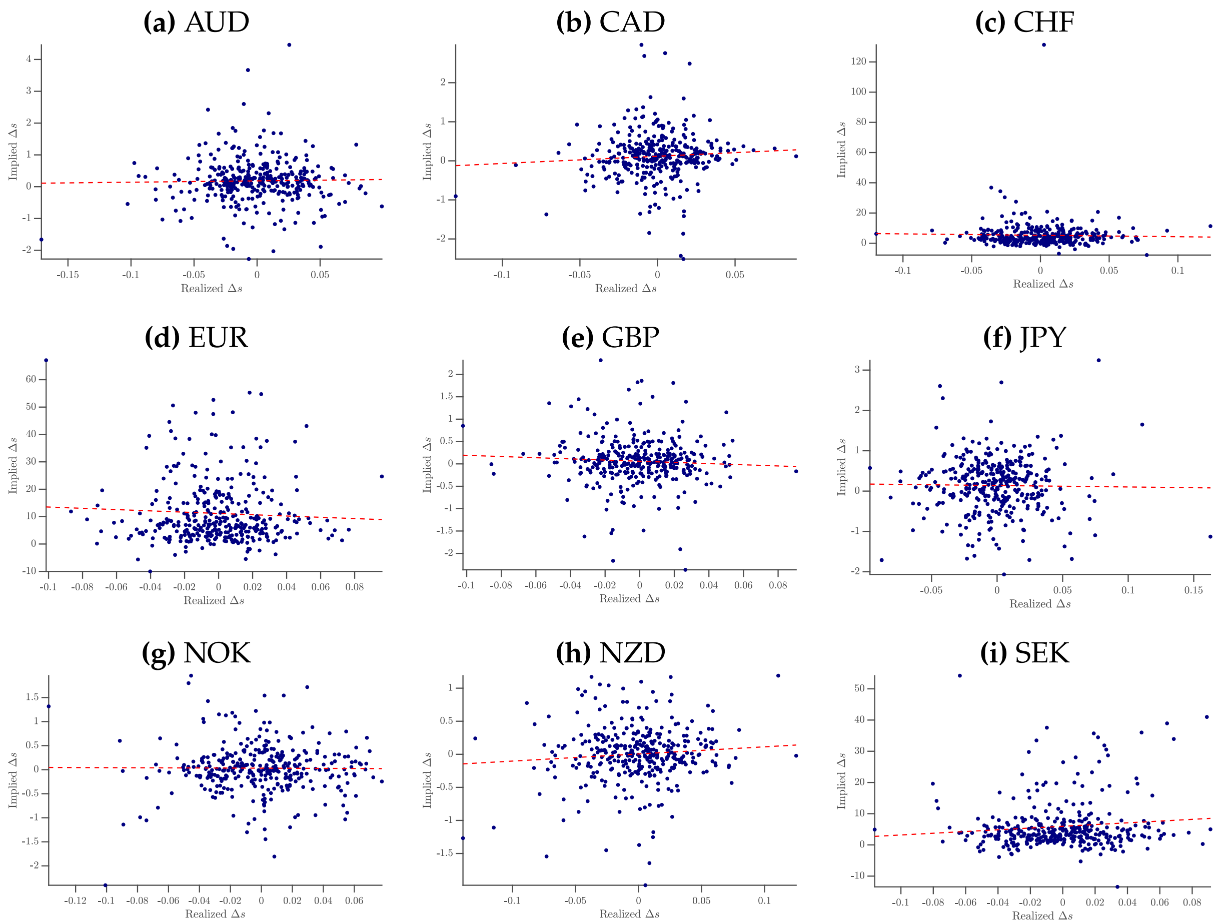

Figure 6: Model-implied vs. Realized Spot Returns

The figure shows the realized (x-axis) and the model-implied (y-axis) exchange rates returns for each of the nine currencies. The red-dashed line corresponds to the linear regression fit. The model-implied spot rates are calculated based on the differences in log stochastic discount factors between the U.S. and the foreign country. Each scatter point represents a monthly observation. The sample period is January 1995 to January 2024.

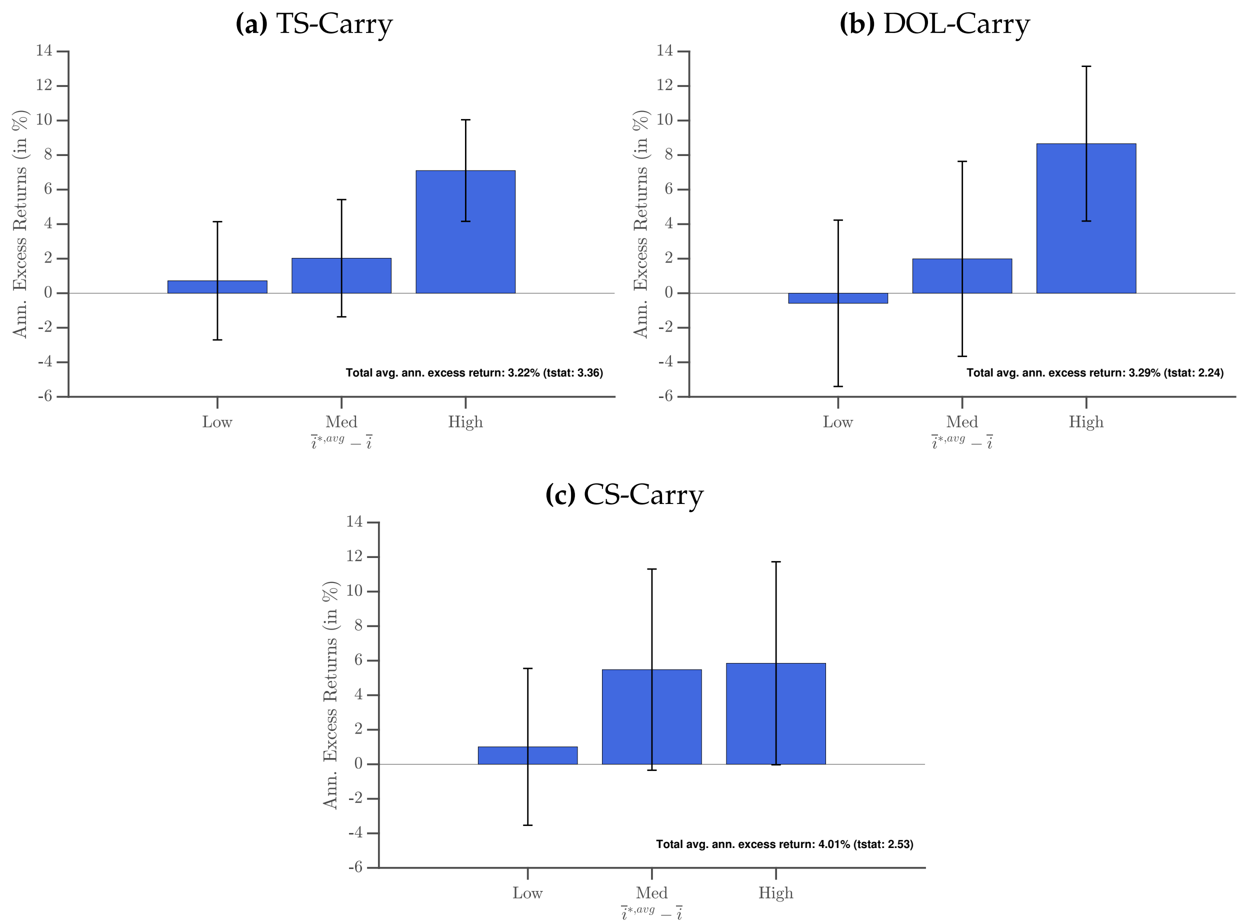

Figure 7: Performance of Trading Strategies and $\bar{i}^{*,avg}$

The figure shows the average annualized excess returns from the trading strategy time-series (TS) carry in Panel (a), dollar (DOL) carry in Panel (b) and the cross-sectional (CS) carry in Panel (c), respectively, in three different sub-samples defined by difference between $\bar{i}^{*,avg}$), the average of the $\bar{i}^{*}$ estimates in a given period across currencies and the US estimate $\bar{i}$. Error bars refer to 10% confidence intervals. Values in the bottom right corner show average annualized returns and corresponding period over the entire sample. The sample period is January 1995 to January 2024.

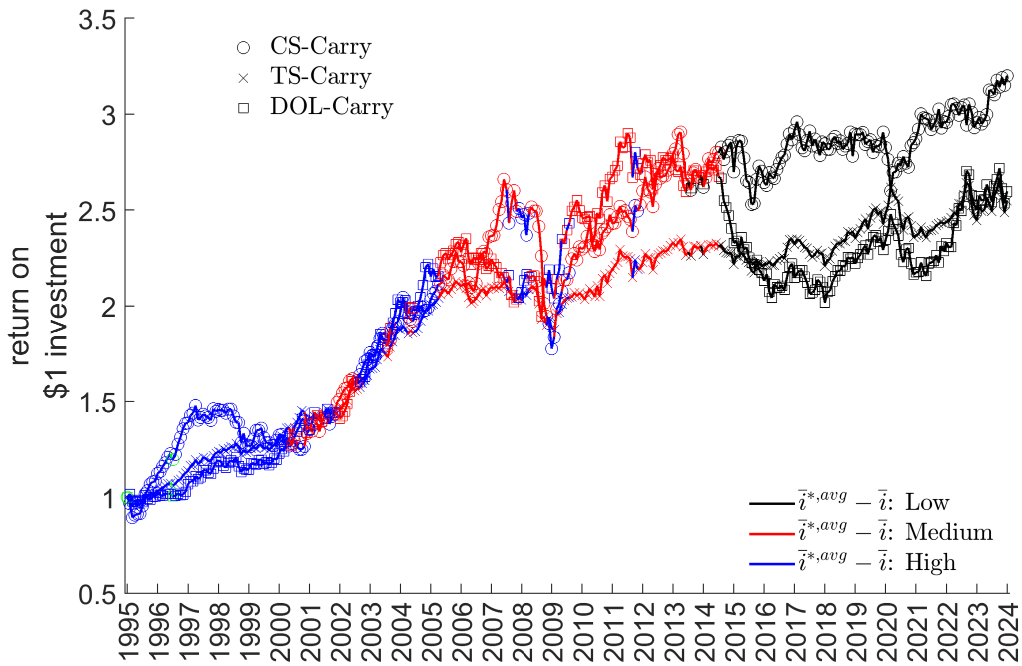

Figure 8: Total Return Indices

The figure shows total return indices for of cross-sectional carry (circle), time-series carry (crosses), and dollar-carry (squares) trading strategies with an initial investment of one U.S. dollar. Period in black, red, and blue indicate months with low, medium and high differentials between the average of $\bar{i}^*$ across all foreign countries (i.e., $\bar{i}^{*,avg}$) and the United States ($\bar{i}$). The sample period is January 1995 to January 2024.

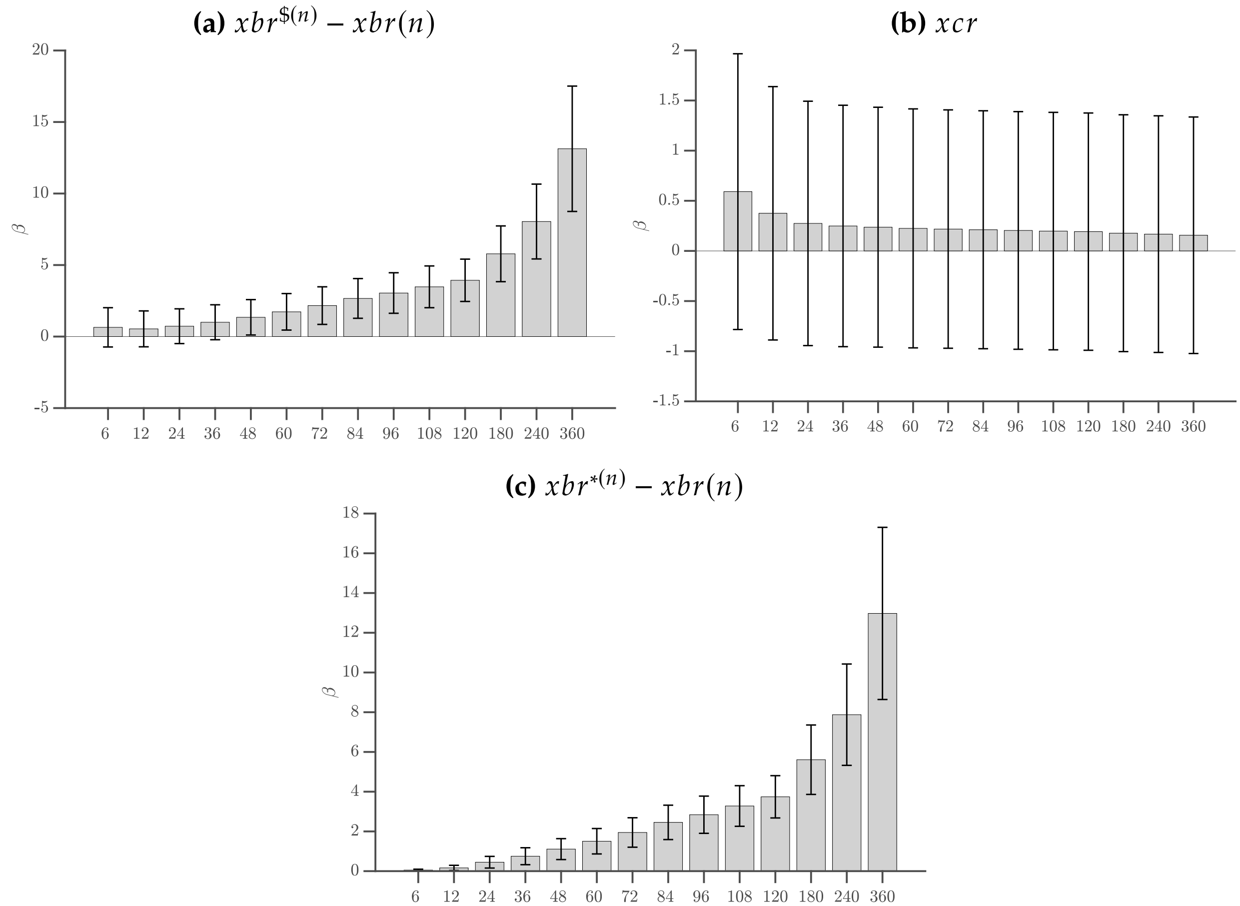

Figure 9: Time Series Predictability Regressions: Term Structure

The figure shows the slope coefficient in predictive panel regressions with currency fixed effects. The predictor is the difference between in the level factors between the US and foreign bond markets. Panel (a) reports results for the excess bond returns differential in dollar $xbr^{\$(n)} - xbr(n)$. Panel (b) reports results for the currency excess returns xcr. Panel (c) reports results for the local currency bond excess returns $xbr^{(n)} - xbr(n)$. The maturity of the left-hand side variables vary between 6-months and 30-years (x-axis). Black bars indicate 10 percent confidence intervals, based on Driscoll-Kray adjusted standard errors. The panel includes nine foreign economies, i.e., Australia, Canada, Switzerland, Euro Area, United Kingdom, Japan, Norway, New Zealand, ans Sweden. The holding period is one quarter and the sample spans January 1999 to January 2024.

Tables

Panel A: Pooled Regressions - Different Maturities						
	$\alpha^{(m)}$	Std. Err	T-stat	P-val	95% - Conf. Int.	N
$3M$	0.040	0.131	0.31	0.760	[-0.217, 0.297]	3,042
$2Y$	0.021	0.130	0.16	0.872	[-0.235, 0.277]	3,042
$5Y$	0.008	0.131	0.06	0.953	[-0.249, 0.265]	3,042
$10Y$	-0.002	0.132	-0.02	0.987	[-0.261, 0.257]	3,042
Panel B: Pooled Regressions with Fixed Effects- 10-years						
	Estimate	Std. Err	T-stat	P-val	95% - Conf. Int.	N
α^{AUD}	-0.023	0.194	-0.12	0.905	[-0.405 ; 0.359]	
α^{CAD}	-0.030	0.124	-0.24	0.811	[-0.273 ; 0.214]	
α^{CHF}	-0.137	0.146	-0.94	0.350	[-0.425 ; 0.151]	
α^{EUR}	0.031	0.155	0.2	0.842	[-0.274 ; 0.335]	
α^{GBP}	0.019	0.142	0.13	0.893	[-0.260 ; 0.298]	3,042
α^{JPY}	0.044	0.179	0.25	0.806	[-0.308 ; 0.396]	
α^{NOK}	0.075	0.175	0.43	0.670	[-0.270 ; 0.419]	
α^{NZD}	-0.073	0.198	-0.37	0.712	[-0.463 ; 0.317]	
α^{SEK}	0.078	0.175	0.45	0.656	[-0.266 ; 0.422]	

Table 1: Regression Analysis: Yield Differential and Currency Excess Returns
The table reports in Panel A results of the pooled panel regression $\xi^{(m)}_{i,t+1} = \alpha^{(m)} + \varepsilon_{i,t+1}$ where $\xi^{(m)}_{i,t+1} = y^{(i,m)}_t - y^{(m)}_t - xcr_{i,t+1}$ refers to the yield differential with maturity m and short-term currency excess returns (xcr) of currency i. In Panel B, results refer to a pooled regression with individual currency fixed effects,

$$\xi^{(m)}_{i,t+1} = \sum_i^N \alpha^{(m)}_i D^i + \varepsilon_{i,t+1}$$

where D^i refers to a dummy variable for currency i and α_i is the corresponding intercept. We compute Driscoll-Kraay standard errors to account for cross-currency correlation, heteroscedasticity, and autocorrelation of the error term. The sample of currencies comprises the G9 currencies vis-à-vis the U.S. dollar: the Australian dollar (AUD), the Canadian dollar (CAD), the Swiss franc (CHF), the euro (EUR), the British pound (GBP), the Japanese yen (JPY), the Norwegian krone (NOK), the New Zealand dollar (NZD), and the Swedish krona (SEK). The sample period is January 1995 to March 2023 (339 monthly observations.)

Panel A: Summary Statistics

	Mean	Median	Std. Dev.	Min	Max	ACorr	ΔFFK (2024)
i^*							
FFK (2024)	3.22	3.01	1.00	2.03	5.05	0.98	
CR (2012)	3.13	2.90	1.01	1.18	5.24	0.96	0.10*
CR (2016)	3.15	2.87	0.99	1.24	5.28	0.95	0.12**
KW (2005)	3.57	3.74	0.84	1.83	5.18	0.94	0.34***
r^*							
HLW (2023)	1.90	1.32	1.03	0.57	3.85	0.98	-1.32***
HT (2014)	1.06	1.04	0.37	0.36	1.88	0.93	-1.90***
LM (2015)	1.52	1.47	0.62	-0.35	2.61	0.83	-1.70***

Panel B: Correlations

	i^*				r^*		
	FFK (2024)	CR (2012)	CR (2016)	KW (2005)	HLW (2023)	HT (2014)	LMs (2015)
i^*							
FFK (2024)	1.00						
CR (2012)	0.82	1.00					
CR (2016)	0.86	0.98	1.00				
KW (2005)	0.87	0.74	0.79	1.00			
r^*							
HLW (2023)	0.93	0.65	0.73	0.78	1.00		
HT (2014)	0.88	0.75	0.83	0.95	0.84	1.00	
LM (2015)	0.65	0.54	0.61	0.54	0.67	0.63	1.00

Table 2: Summary Statistics: United States Trend Estimates
The table reports summary statistics of the interest rates trend for the US. Panel A shows the (annualized) mean, median, standard deviation, minimum and maximum values, and the first-order autocorrelation measure (ACorr). ΔFFK (2024) reports the average difference between FFK (2024) and each of the other estimates over the sample period in which both sets of estimates are available. ***, **, * denote significance at the 1-, 5-, and 10-percent level. The average difference are calculated over the sample period when both estimates are available. Panel B shows correlation coefficients between alternative estimates of the trend. The sample period is January 1995 to January 2024.

Panel A: i^*-Differential

	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	SoE^{Avg}
Mean	1.18	0.15	-1.65	-1.01	0.66	-1.62	0.47	2.15	0.00	0.04
t-stat	(356.78)	(5.09)	(-67.27)	(-36.83)	(19.32)	(-73.90)	(26.40)	(106.22)	(20.85)	(3.27)
Median	1.19	0.06	-1.75	-0.98	0.66	-1.78	0.43	2.18	0.00	0.02
z-stat	(18.63)	(2.46)	(-18.63)	(-16.81)	(11.35)	(-18.63)	(15.95)	(18.63)	(13.38)	(2.36)
Std. Dev.	0.06	0.57	0.46	0.51	0.64	0.41	0.33	0.38	0.00	0.21
Minimum.	1.03	-0.71	-2.46	-1.73	-0.49	-2.31	-0.13	1.18	-0.00	-0.32
Maximum.	1.31	2.10	-0.49	0.37	2.32	-0.68	1.14	3.03	0.00	0.67
Skewness	-0.32	1.34	0.91	0.57	0.25	0.49	0.22	-0.42	-0.01	1.17
Kurtosis	2.34	4.78	3.38	2.74	2.89	2.06	2.00	3.19	2.19	4.43

Panel B: i^*-Correlations

	USD	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	SoE^{Avg}
USD	1.00										
AUD	1.00	1.00									
CAD	0.95	0.96	1.00								
CHF	0.98	0.98	0.96	1.00							
EUR	0.99	0.99	0.97	0.97	1.00						
GBP	0.94	0.95	0.93	0.92	0.96	1.00					
JPY	0.94	0.93	0.88	0.94	0.92	0.85	1.00				
NOK	0.99	0.98	0.93	0.97	0.97	0.90	0.96	1.00			
NZD	0.95	0.96	0.92	0.91	0.96	0.96	0.83	0.92	1.00		
SEK	1.00	1.00	0.95	0.98	0.99	0.94	0.94	0.99	0.95	1.00	
SOE^{Avg}	0.99	1.00	0.97	0.98	1.00	0.96	0.93	0.98	0.96	0.99	1.00

Table 3: Summary Statistics: Global i^*
The table reports summary statistics of i^*-differentials at the country level in Panel A, showing the (annualized) mean and median with the corresponding t- and $z-$ statistics in parentheses, volatility, minimum and maximum values, as well as skewness and kurtosis. Panel B shows correlation coefficients between global i^* estimates. The column "SoE^{Avg}" refers to the unconditional average of all SoE's i^*-estimates. Values are expressed in percentage. The sample period is January 1995 to January 2024.

Panel A: $rx^{*,\$} - rx$										
	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	Panel
β	17.86	11.89	10.59	4.78	7.22	18.04	29.26	31.01	14.26	13.13
	(2.93)	(2.36)	(2.14)	(1.95)	(2.00)	(3.30)	(3.50)	(3.97)	(5.83)	(4.91)
R^2 (in %)	9.32	6.59	2.92	1.75	2.35	8.61	15.20	12.54	16.48	7.52
N	116	116	116	116	116	116	116	116	116	1044

Panel B: rx^{FX}										
	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	Panel
β	2.39	1.60	-2.67	-0.71	0.49	-6.87	5.11	3.04	1.46	0.16
	(1.13)	(1.45)	(-1.73)	(-0.64)	(0.47)	(-3.44)	(1.99)	(0.81)	(1.52)	(0.22)
R^2 (in %)	-0.24	0.15	1.05	-0.65	-0.77	6.52	2.13	-0.27	0.03	0.01
N	116	116	116	116	116	116	116	116	116	1044

Panel C: $rx^{*} - rx$										
	AUD	CAD	CHF	EUR	GBP	JPY	NOK	NZD	SEK	Panel
β	15.47	10.29	13.26	5.49	6.72	24.92	24.16	27.97	12.80	12.97
	(2.63)	(2.20)	(2.82)	(2.54)	(2.23)	(3.80)	(3.32)	(3.97)	(5.49)	(4.91)
R^2 (in %)	9.26	7.45	5.83	3.76	3.41	15.04	14.85	14.26	18.85	9.32
N	116	116	116	116	116	116	116	116	116	1044

Table 4: Time Series Predictability Regressions: Level Differentials
The figure shows the slope coefficient of panel regressions with the bond dollar return difference ($rx^{\$,*} - rx$), currency excess returns (rx^{FX}), or local currency bond excess returns ($rx^{*} - rx$) on the difference in level factors between foreign (small open economy) and the domestic (United States) economy. The maturity of the left-hand side variables are 30-years. Numbers in parentheses show t-statistics based on Newey-West or Driscoll-Kray adjusted standard errors. The holding period is one quarter and the sample spans January 1999 to January 2024.

Internet Appendix

AI Proof of Proposition 2

$$M_{t+1}^{\mathcal{T}} = \frac{\Lambda_{t+1}^{\mathcal{T}}}{\Lambda_t^{\mathcal{T}}}$$

$$\Lambda_t^{\mathcal{T}} \equiv \lim_{n \longrightarrow \infty} \frac{\delta^{t+n}}{P_{\zeta_t}^{(n)}} = \delta^t \lim_{n \longrightarrow \infty} \exp(n \ln \delta - A_{\zeta,n} - B_{\zeta,n}^{\top} \zeta_t),$$

where

$$\begin{aligned}
B_{\zeta,n}^{\top} &= B_{\zeta,\infty}^{\top} \left(I - \left(\phi^{\mathbb{Q}}\right)^n\right), \\
B_{\zeta,\infty}^{\top} &= -\delta_{\zeta}^{\top} \left(I - \phi^{\mathbb{Q}}\right)^{-1}, \\
A_{\zeta,n} &= \left(\sum_{j=0}^{n-1} B_{\zeta,j}^{\top}\right) \mu^{\mathbb{Q}} + \frac{1}{2} \sum_{j=0}^{n-1} \left(B_{\zeta,j}^{\top} \Omega_{\zeta} B_{\zeta,j}\right) - n \delta_{0,\zeta}.
\end{aligned}$$

We have,

$$\begin{aligned}
n \ln \delta - A_{\zeta,n} &= n \left(\ln \delta + \delta_{0,\zeta} - B_{\zeta,\infty}^{\top} \mu^{\mathbb{Q}}\right) \\
&\quad + B_{\zeta,\infty}^{\top} \left(I - \left(\phi^{\mathbb{Q}}\right)^n\right) \left(I - \phi^{\mathbb{Q}}\right)^{-1} \mu^{\mathbb{Q}} - \frac{1}{2} \sum_{j=0}^{n-1} \left(B_{\zeta,j}^{\top} \Omega_{\zeta} B_{\zeta,j}\right)
\end{aligned}$$

and using $B_{\zeta,j}^{\top} - B_{\zeta,\infty}^{\top} = -B_{\zeta,\infty}^{\top} \left(\phi^{\mathbb{Q}}\right)^j$,

$$\begin{aligned}
\sum_{j=0}^{n-1} \left(B_{\zeta,j}^{\top} \Omega_{\zeta} B_{\zeta,j}\right) &= \sum_{j=0}^{n-1} B_{\zeta,\infty}^{\top} \Omega_{\zeta} B_{\zeta,\infty} + \sum_{j=0}^{n-1} \left(B_{\zeta,j} - B_{\zeta,\infty}\right)^{\top} \Omega_{\zeta} \left(B_{\zeta,j} - B_{\zeta,\infty}\right) \\
&\quad + 2 \sum_{j=0}^{n-1} \left(B_{\zeta,j} - B_{\zeta,\infty}\right)^{\top} \Omega_{\zeta} B_{\zeta,\infty} \\
&= n B_{\zeta,\infty}^{\top} \Omega_{\zeta} B_{\zeta,\infty} + B_{\zeta,\infty}^{\top} \left[\sum_{j=0}^{n-1} \left(\phi^{\mathbb{Q}}\right)^j \Omega_{\zeta} \left(\left(\phi^{\mathbb{Q}}\right)^j\right)^{\top}\right] B_{\zeta,\infty}
\end{aligned}$$

$$
\begin{aligned}
&\quad -2B_{\zeta,\infty}^{\top}\left[\sum_{j=0}^{n-1}\left(\phi^{\mathbb{Q}}\right)^{j}\right]\Omega_{\zeta}B_{\zeta,\infty} \\
&= nB_{\zeta,\infty}^{\top}\Omega_{\zeta}B_{\zeta,\infty}+B_{\zeta,\infty}^{\top}\left[\sum_{j=0}^{n-1}\left(\phi^{\mathbb{Q}}\right)^{j}\Omega_{\zeta}\left(\left(\phi^{\mathbb{Q}}\right)^{j}\right)^{\top}\right]B_{\zeta,\infty} \\
&\quad -2B_{\zeta,\infty}^{\top}\left(I-\left(\phi^{\mathbb{Q}}\right)^{n}\right)\left(I-\phi^{\mathbb{Q}}\right)^{-1}\Omega_{\zeta}B_{\zeta,\infty},
\end{aligned}
$$

which implies that,

$$
\begin{aligned}
n\ln\delta-A_{\zeta,n} &= n\left(\ln\delta+\delta_{0,\zeta}-B_{\zeta,\infty}^{\top}\mu^{\mathbb{Q}}-\frac{1}{2}B_{\zeta,\infty}^{\top}\Omega_{\zeta}B_{\zeta,\infty}\right) \\
&\quad +B_{\zeta,\infty}^{\top}\left(I-\left(\phi^{\mathbb{Q}}\right)^{n}\right)\left(I-\phi^{\mathbb{Q}}\right)^{-1}\left(\mu^{\mathbb{Q}}+\Omega_{\zeta}B_{\zeta,\infty}\right) \\
&\quad -\frac{1}{2}B_{\zeta,\infty}^{\top}\left[\sum_{j=0}^{n-1}\left(\phi^{\mathbb{Q}}\right)^{j}\Omega_{\zeta}\left(\left(\phi^{\mathbb{Q}}\right)^{j}\right)^{\top}\right]B_{\zeta,\infty}.
\end{aligned}
$$

Hence, $n\ln\delta-A_{\zeta,n}$ converges if and only if

$$
\ln\delta+\delta_{0,\zeta}-B_{\zeta,\infty}^{\top}\mu^{\mathbb{Q}}-\frac{1}{2}B_{\zeta,\infty}^{\top}\Omega_{\zeta}B_{\zeta,\infty}=0,
$$

which implies that:

$$
\ln\delta=-\delta_{0,\zeta}+B_{\zeta,\infty}^{\top}\mu^{\mathbb{Q}}+\frac{1}{2}B_{\zeta,\infty}^{\top}\Omega_{\zeta}B_{\zeta,\infty},
$$

and

$$
\begin{aligned}
\tilde{\lambda}_{0,\zeta}\equiv\lim_{n\Longrightarrow\infty} n\ln\delta-A_{\zeta,n} &= B_{\zeta,\infty}^{\top}\left(I-\phi^{\mathbb{Q}}\right)^{-1}\left(\mu^{\mathbb{Q}}+\Omega_{\zeta}B_{\zeta,\infty}\right) \\
&\quad -\frac{1}{2}B_{\zeta,\infty}^{\top}\left[\sum_{j=0}^{\infty}\left(\phi^{\mathbb{Q}}\right)^{j}\Omega_{\zeta}\left(\left(\phi^{\mathbb{Q}}\right)^{j}\right)^{\top}\right]B_{\zeta,\infty}
\end{aligned}
$$

Thus

$$
\Lambda_t^{\mathcal{T}}=\delta^{t}\exp(\tilde{\lambda}_{0,\zeta}-B_{\zeta,\infty}^{\top}\zeta_t),
$$

and

$$
M_{t+1}^{\mathcal{T}}=\frac{\Lambda_{t+1}^{\mathcal{T}}}{\Lambda_t^{\mathcal{T}}}=\exp(\ln(\delta)-B_{\zeta,\infty}^{\top}\Delta\zeta_{t+1}).
$$

This implies that:

$$Var_t\left[\ln\left(M^{\mathcal{T}}_{t+1}\right)\right] = B^{\top}_{\zeta,\infty}\Omega_{\zeta}B_{\zeta,\infty},$$

and

$$\ln\delta = -\delta_{0,\zeta} + B^{\top}_{\zeta,\infty}\mu^{\mathbb{Q}} + \frac{1}{2}Var_t\left[\ln\left(M^{\mathcal{T}}_{t+1}\right)\right].$$

$$\begin{aligned}
M^{\mathcal{T}}_{t+1} &= \exp\left(\ln(\delta) - B^{\top}_{\zeta,\infty}\Delta\zeta_{t+1}\right) \\
&= \exp\left(-\delta_{0,\zeta} + B^{\top}_{\zeta,\infty}\mu^{\mathbb{Q}} + \frac{1}{2}Var_t\left[\ln\left(M^{\mathcal{T}}_{t+1}\right)\right] - B^{\top}_{\zeta,\infty}\Delta\zeta_{t+1}\right) \\
&= \exp\left(-i_{\zeta,t} + \frac{1}{2}Var_t\left[\ln\left(M^{\mathcal{T}}_{t+1}\right)\right] - B^{\top}_{\zeta,\infty}\left(\zeta_{t+1} - E^{\mathbb{Q}}_t\left[\zeta_{t+1}\right]\right)\right),
\end{aligned}$$

hence

$$E^{\mathbb{Q}}_t\left[M^{\mathcal{T}}_{t+1}\right] = \exp\left(-i_{\zeta,t} + Var_t\left[\ln\left(M^{\mathcal{T}}_{t+1}\right)\right]\right).$$

$$\begin{aligned}
E_t\left[M^{\mathcal{P}}_{t+1}\right] &= E_t\left[\frac{M_{t+1}}{M^{\mathcal{T}}_{t+1}}\right] = E_t\left[M_{t+1}\right]E^{\mathbb{Q}}_t\left[\frac{1}{M^{\mathcal{T}}_{t+1}}\right] \\
&= E_t\left[M_{t+1}\right]E^{\mathbb{Q}}_t\left[\exp\left(-\ln\left(M^{\mathcal{T}}_{t+1}\right)\right)\right] \\
&= \exp(-i_t)\exp\left(-E^{\mathbb{Q}}_t\left[\ln\left(M^{\mathcal{T}}_{t+1}\right)\right] + \frac{1}{2}Var_t\left[\ln\left(M^{\mathcal{T}}_{t+1}\right)\right]\right) \\
&= \exp(-i_t)\exp\left(-E^{\mathbb{Q}}_t\left[\ln\left(M^{\mathcal{T}}_{t+1}\right)\right] - \frac{1}{2}Var_t\left[\ln\left(M^{\mathcal{T}}_{t+1}\right)\right] + Var_t\left[\ln\left(M^{\mathcal{T}}_{t+1}\right)\right]\right) \\
&= \frac{\exp(-i_t)}{E^{\mathbb{Q}}_t\left[M^{\mathcal{T}}_{t+1}\right]}\exp\left(Var_t\left[\ln\left(M^{\mathcal{T}}_{t+1}\right)\right]\right) = \exp\left(-i_{\tau,t}\right)
\end{aligned}$$

AII Identification and estimation

AII.1 The US model

AII.1.1 Risk-neutral

$$y^{(n)}_t = a_n + \tau_t + b^{\top}_{n,\zeta}\zeta_t$$

where

$$b^{\top}_{n,\zeta} \equiv \frac{1}{n}B^{\top}_{n,\zeta}$$

where

$$B_{n,\zeta}^{\top} = \tilde{\lambda}_{\zeta}^{\top}\left(I - \left(\phi^{\mathbb{Q}}\right)^{n}\right)$$

and

$$\tilde{\lambda}_{\zeta}^{\top} \equiv \delta_{\zeta}^{\top}\left(I - \phi^{\mathbb{Q}}\right)^{-1}.$$

$$a_n = A_n/n$$

where follows the recurrence,

$$A_{n+1} = A_n + B_{n,\zeta}^{\top}\mu^{\mathbb{Q}} - \frac{1}{2}\left(B_{n,\zeta}^{\top}\Omega_{\zeta}B_{n,\zeta}\right) - \frac{n^2}{2}\Omega_{\tau} + \delta_0$$

with initial condition $A_0 = 0$, and $\delta_0 = \delta_{0,\tau} + \delta_{0,\zeta}$

We fix $\delta_{0,\tau} = \delta_{0,\zeta} = 0$, $\delta_{\tau} = 1$,and $\tilde{\lambda}_{\zeta}$ to a vector of 1.

Estimation of $\phi^{\mathbb{Q}}$ and filtering of $\Delta\zeta_t$ We have

$$\Delta y_t^{(n)} = \Delta\tau_t + b_{n,\zeta}^{\top}\Delta\zeta_t$$

and

$$\Delta\left(y_t^{(n)} - y_t^{(n_1)}\right) = \left(b_{n,\zeta}^{\top} - b_{n_1,\zeta}^{\top}\right)\Delta\zeta_t$$

hence

$$\Delta\left(Y_t - y_t^{(n_1)}\right) = \begin{pmatrix} b_{n_2,\zeta}^{\top} - b_{n_1,\zeta}^{\top} \\ b_{n_3,\zeta}^{\top} - b_{n_1,\zeta}^{\top} \\ \vdots \\ b_{n_J,\zeta}^{\top} - b_{n_1,\zeta}^{\top} \end{pmatrix}\Delta\zeta_t.$$

Next, we denote by W a $N_{\zeta} \times J - 1$ matrix of coefficient, representing the first PCAs of $Y_t - y_t^{(n_1)}$

$$W\Delta\left(Y_t - y_t^{(n_1)}\right) = W\begin{pmatrix} b_{n_2,\zeta}^{\top} - b_{n_1,\zeta}^{\top} \\ b_{n_3,\zeta}^{\top} - b_{n_1,\zeta}^{\top} \\ \vdots \\ b_{n_J,\zeta}^{\top} - b_{n_1,\zeta}^{\top} \end{pmatrix}\Delta\zeta_t$$

and $\Delta\zeta_t$ is filter as following:

$$\Delta\zeta_t = \left\{ W \begin{pmatrix} b_{n_2,\zeta}^\top - b_{n_1,\zeta}^\top \\ b_{n_3,\zeta}^\top - b_{n_1,\zeta}^\top \\ \vdots \\ b_{n_J,\zeta}^\top - b_{n_1,\zeta}^\top \end{pmatrix} \right\}^{-1} W\Delta\left(Y_t - y_t^{(n_1)}\right)$$

Next, we denote by W_e a $(J-1-N_\zeta)\times J-1$ matrix of coefficient representing the other PCAs, orthogonal to W. We estimate $\phi^{\mathbb{Q}}$ by minimizing the wedge between observed and model implied value for $W_e\Delta\left(Y_t - y_t^{(n_1)}\right)$.

Estimation of ζ_1, τ_1, Ω_ζ, Ω_τ, $\mu^{\mathbb{Q}}$ and the filtering of τ_t Define

$$\bar{\zeta}_t \equiv \sum_{s=2}^{t} \Delta\zeta_s$$

and we estimate Ω_ζ to the sample covariance of $\bar{\zeta}_t$, that is

$$\Omega_\zeta = \frac{\sum_{t=2}^{T}\left(\bar{\zeta}_t - \hat{\zeta}_T\right)\left(\bar{\zeta}_t - \hat{\zeta}_T\right)^\top}{T-1}$$

where

$$\hat{\zeta}_T \equiv \frac{\sum_{t=2}^{T}\bar{\zeta}_t}{T-1}.$$

$$y_t^{(n)} = a_n + \tau_t + b_{n,\zeta}^\top\zeta_t,$$

hence

$$y_t^{(n)} - y_1^{(n)} = \bar{\tau}_t + b_{n,\zeta}^\top\bar{\zeta}_t$$

where

$$\bar{\tau}_t \equiv \tau_t - \tau_1$$

we filter $\bar{\tau}_t$ as following

$$\bar{\tau}_t = \left(\frac{1}{J}\sum_{j=1}^{J}\left(y_t^{(n_j)} - y_1^{(n_j)}\right)\right) - \left(\frac{1}{J}\sum_{j=1}^{J} b_{n_j,\zeta}^\top\right)\bar{\zeta}_t$$

and estimate

$$\Omega_\tau = \frac{\sum_{t=2}^{T}\left(\bar{\tau}_t - \hat{\tau}_T\right)^2}{T-1}$$

where

$$\hat{\tau}_T \equiv \frac{\sum_{t=2}^{T} \bar{\tau}_t}{T-1}.$$

Given that,

$$\begin{aligned}
\zeta_t &= \zeta_1 + \bar{\zeta}_t \\
\tau_t &= \tau_1 + \bar{\tau}_t
\end{aligned}$$

our remaining task is to estimate ζ_1, and τ_1, to do that, first

$$y_t^{(n)} - y_t^{(n_1)} = a_n - a_{n_1} + \left(b_{n,\zeta}^\top - b_{n_1,\zeta}^\top\right)\zeta_t$$

$$\begin{aligned}
y_t^{(n)} - y_t^{(n_1)} &= a_n - a_{n_1} + \left(b_{n,\zeta}^\top - b_{n_1,\zeta}^\top\right)\left(\zeta_1 + \bar{\zeta}_t\right) \\
&= a_n - a_{n_1} + \left(b_{n,\zeta}^\top - b_{n_1,\zeta}^\top\right)\zeta_1 + \left(b_{n,\zeta}^\top - b_{n_1,\zeta}^\top\right)\bar{\zeta}_t
\end{aligned}$$

hence

$$y_t^{(n)} - y_t^{(n_1)} - \left(b_{n,\zeta}^\top - b_{n_1,\zeta}^\top\right)\bar{\zeta}_t = a_n - a_{n_1} + \left(b_{n,\zeta}^\top - b_{n_1,\zeta}^\top\right)\zeta_1$$

denote

$$z_t^{(n)} \equiv y_t^{(n)} - y_t^{(n_1)} - \left(b_{n,\zeta}^\top - b_{n_1,\zeta}^\top\right)\bar{\zeta}_t$$

hence

$$\begin{pmatrix} z_t^{(n_2)} \\ z_t^{(n_3)} \\ \vdots \\ z_t^{(n_J)} \end{pmatrix} = \begin{pmatrix} a_{n_2} - a_{n_1} \\ a_{n_3} - a_{n_1} \\ \vdots \\ a_{n_J} - a_{n_1} \end{pmatrix} + \begin{pmatrix} b_{n_2,\zeta}^\top - b_{n_1,\zeta}^\top \\ b_{n_3,\zeta}^\top - b_{n_1,\zeta}^\top \\ \vdots \\ b_{n_J,\zeta}^\top - b_{n_1,\zeta}^\top \end{pmatrix}\zeta_1$$

hence

$$\begin{pmatrix} z_t^{(n_2)} \\ z_t^{(n_3)} \\ \vdots \\ z_t^{(n_J)} \end{pmatrix} = \begin{pmatrix} a_{n_2} \\ a_{n_3} \\ \vdots \\ a_{n_J} \end{pmatrix} - a_{n_1} + \begin{pmatrix} b_{n_2,\zeta}^\top - b_{n_1,\zeta}^\top \\ b_{n_3,\zeta}^\top - b_{n_1,\zeta}^\top \\ \vdots \\ b_{n_J,\zeta}^\top - b_{n_1,\zeta}^\top \end{pmatrix}\zeta_1.$$

Let us denote by W_z a $N_\zeta \times J - 1$ matrix of coefficient, representing the first PCAs of $z_t^{(n_J)}$, we have

$$W_z \begin{pmatrix} z_t^{(n_2)} \\ z_t^{(n_3)} \\ \vdots \\ z_t^{(n_J)} \end{pmatrix} = W_z \begin{pmatrix} a_{n_2} \\ a_{n_3} \\ \vdots \\ a_{n_J} \end{pmatrix} - a_{n_1} W_z \iota_{J-1} + W_z \begin{pmatrix} b_{n_2,\zeta}^\top - b_{n_1,\zeta}^\top \\ b_{n_3,\zeta}^\top - b_{n_1,\zeta}^\top \\ \vdots \\ b_{n_J,\zeta}^\top - b_{n_1,\zeta}^\top \end{pmatrix} \zeta_1$$

$$\hat{z} = A_z - a_{n_1} W_z \iota_{J-1} + B_z \zeta_1$$

where

$$\hat{z} = \frac{1}{T-1} W_z \begin{pmatrix} \sum_{t=2}^{T} z_t^{(n_2)} \\ \sum_{t=2}^{T} z_t^{(n_3)} \\ \vdots \\ \sum_{t=2}^{T} z_t^{(n_J)} \end{pmatrix} ;;; B_z \equiv W_z \begin{pmatrix} b_{n_2,\zeta}^\top - b_{n_1,\zeta}^\top \\ b_{n_3,\zeta}^\top - b_{n_1,\zeta}^\top \\ \vdots \\ b_{n_J,\zeta}^\top - b_{n_1,\zeta}^\top \end{pmatrix} ;;; A_z \equiv W_z \begin{pmatrix} a_{n_2} \\ a_{n_3} \\ \vdots \\ a_{n_J} \end{pmatrix}$$

hence

$$\zeta_1 = B_z^{-1} \left(\hat{z} - A_z + a_{n_1} \left(W_z \iota_{J-1} \right) \right)$$

$$y_1^{(n)} = a_n + \tau_1 + b_{n,\zeta}^\top \zeta_1$$

hence

$$\left(\frac{1}{J} \sum_{j=1}^{J} y_1^{(n_j)} \right) = \left(\frac{1}{J} \sum_{j=1}^{J} a_{n_j} \right) + \tau_1 + \left(\frac{1}{J} \sum_{j=1}^{J} b_{n_j,\zeta}^\top \right) \zeta_1$$

hence

$$\tau_1 = \left(\frac{1}{J} \sum_{j=1}^{J} y_1^{(n_j)} \right) - \left(\frac{1}{J} \sum_{j=1}^{J} b_{n_j,\zeta}^\top \right) \zeta_1 - \left(\frac{1}{J} \sum_{j=1}^{J} a_{n_j} \right).$$

We estimate $\mu^{\mathbb{Q}}$ by minimizing the gap between observed yield $\left(y_t^{(n)} \right)$ and model implied.

AII.1.2 P-Dynamic

$$x_t \equiv \begin{pmatrix} \tau_t \\ \zeta_t \end{pmatrix}$$

$$\varepsilon_{x,t+1} = x_{t+1} - \mathcal{E}_{x,t}$$

$$\tau_t = \bar{\tau}_t + \tilde{\tau}_t.$$

We assume the following dynamic for $\bar{\tau}_t$

$$\bar{\tau}_{t+1} = \bar{\tau}_t + \bar{\sigma}^\top \varepsilon_{x,t+1}.$$

We also assume that both the level factor τ_t and the slope and curvature share the same trend, that is:

$$\tilde{\zeta}_t \equiv \zeta_t - \bar{\tau}_t \gamma,$$

and stack together the stationary components of the yield curve in a vector denoted by $\tilde{x}_t$,

$$\tilde{x}_t \equiv \begin{pmatrix} \tilde{\tau}_t \\ \tilde{\zeta}_t \end{pmatrix}.$$

We denote the time t expectation of $\tilde{x}_{t+1}$ by $\tilde{\mathcal{E}}_{x,t}$, that is

$$\tilde{\mathcal{E}}_{x,t} \equiv E_t \left[\tilde{x}_{t+1}\right],$$

and assume the following dynamic for $\tilde{\mathcal{E}}_{x,t}$,

$$\tilde{\mathcal{E}}_{x,t+1} = \mu + \phi \tilde{\mathcal{E}}_{x,t} + \psi \varepsilon_{x,t+1}.$$

We estimate physical parameters $\bar{\sigma}$, γ, μ, ϕ and ψ by maximizing the Gaussian likelihood for $\varepsilon_{x,t}$, assuming $\varepsilon_{x,t} \sim N\left(0, \frac{1}{T}\sum_{t=1}^{T} \varepsilon_{x,t}\varepsilon_{x,t}^\top\right)$.

AII.2 The SoE model

AII.2.1 Risk-neutral The discussions are identical to the discussion of the risk-neutral model for the US. Except, we model the cross country interest rate differential, that is we model $\Delta_c y_t^{(n)} \equiv y_t^{(n)} - y_t^{(n,*)}$, where $y_t^{(n,*)}$ is the SoE nominal yield.

$$\Delta_c y_t^{(n)} = a_n^* + \tau_t^* + b_{n,\zeta^*}^\top \zeta_t^*$$

$$b_{n,\zeta^*}^\top = (1/n)\, B_{n,\zeta^*}^\top$$

where

$$B_{n,\zeta^*}^\top \equiv \tilde{\lambda}_{\zeta^*}^\top \left(I - \left(\phi_*^{\mathbb{Q}}\right)^n\right)$$

with

$$\tilde{\lambda}_{\zeta^*}^\top \equiv \delta_{\zeta^*}^\top \left(I - \phi_*^{\mathbb{Q}}\right)^{-1}.$$

$$a_n^* = (1/n)\, A_n^*$$

$$A^*_{n+1} = A^*_n + B^\top_{n,\zeta^*}\mu^{\mathbb{Q}}_* - \frac{1}{2}\left(B^\top_{n,\zeta^*}\Omega_{\zeta^*}B_{n,\zeta^*}\right) - \frac{n^2}{2}\Omega_{\tau^*} + \delta^*_0$$

with initial condition $A^*_0 = 0$, and $\delta^*_0 = -\tilde{\lambda}^\top_{\zeta^*}\mu^{\mathbb{Q}}_* + \frac{1}{2}\tilde{\lambda}^\top_{\zeta^*}\Omega_{\zeta^*}\tilde{\lambda}_{\zeta^*}$.

AII.2.2 Cyclical component of the exchange rate

$$\begin{aligned} M^{\mathcal{T}}_{t+1} &= \exp(-\delta_{0,\zeta} - \tilde{\lambda}^\top_\zeta\mu^{\mathbb{Q}} + \frac{1}{2}\tilde{\lambda}^\top_\zeta\Omega_\zeta\tilde{\lambda}_\zeta + \tilde{\lambda}^\top_\zeta\Delta\zeta_{t+1}). \\ \frac{M^{*\mathcal{T}}_{t+1}}{M^{\mathcal{T}}_{t+1}} &= \exp(-\delta^*_0 - \tilde{\lambda}^\top_{\zeta^*}\mu^{\mathbb{Q}}_* + \frac{1}{2}\tilde{\lambda}^\top_{\zeta^*}\Omega_{\zeta^*}\tilde{\lambda}_{\zeta^*} + \tilde{\lambda}^\top_{\zeta^*}\Delta\zeta^*_{t+1}), \end{aligned}$$

hence

$$\begin{aligned} \Delta\tilde{s}_{t+1} &= \Delta_c \ln M^{\mathcal{T}}_{t+1} = -\delta^*_0 - \tilde{\lambda}^\top_{\zeta^*}\mu^{\mathbb{Q}}_* + \frac{1}{2}\tilde{\lambda}^\top_{\zeta^*}\Omega_{\zeta^*}\tilde{\lambda}_{\zeta^*} + \tilde{\lambda}^\top_{\zeta^*}\Delta\zeta^*_{t+1} \\ &= \tilde{\lambda}^\top_{\zeta^*}\Delta\zeta^*_{t+1}, \end{aligned}$$

hence

$$\tilde{s}_t = \tilde{\lambda}^\top_{\zeta^*}\zeta^*_t,$$

and thus to guaranty the stationarity of $\tilde{s}_t$, we will assume that ζ^*_t is also stationary under the physical probability measure.

AII.2.3 Physical dynamics

$$x^*_t \equiv \begin{pmatrix} \tau^*_t \\ \zeta^*_t \end{pmatrix}$$

In addition, the vector z_t stacks together the excess currency returns with the yields spreads factors, as follows:

$$z_t \equiv \begin{bmatrix} xcr_t \\ x^*_t \end{bmatrix},$$

because the dynamics of the exchange rate and yield spreads are connected by the absence of arbitrage, and they have correlated innovations, given by:

$$\varepsilon_{z,t+1} = z_{t+1} - \mathcal{E}_{z,t},$$

in particular, focussing on the first element of z_t, we have

$$\varepsilon_{xcr,t+1} = xcr_{t+1} - \mathcal{E}_{xcr,t},$$

where $\mathcal{E}_{xcr,t}$, the first element of $\mathcal{E}_{z,t}$ is the exchange rate risk premium. We decompose the level factor of the cross-country differential between the nominal yield curve as follows

$$\tau_t^* = \bar{\tau}_t^* + \tilde{\tau}_t^*,$$

and assume the following dynamic for $\bar{\tau}_t^*$

$$\bar{\tau}_{t+1}^* = \bar{\tau}_t^* + \bar{\sigma}_*^\top \varepsilon_{z,t+1}.$$

We also assume that both the level factor τ_t^* and the slope and curvature share the same trend, that is:

$$\tilde{\zeta}_t^* \equiv \zeta_t^* - \bar{\tau}_t^* \gamma^*,$$

and stack together the stationary components of the yield curve in a vector denoted by $\tilde{x}_t^*$,

$$\tilde{x}_t^* \equiv \begin{pmatrix} \tilde{\tau}_t^* \\ \tilde{\zeta}_t^* \end{pmatrix}.$$

As discussed in the previous subsection, a necessary condition for the stationarity of $\tilde{s}_t$ under the physical probability measure is $\gamma^* = 0$, we impose that condition. We denote the time t expectation of $\tilde{x}_{t+1}^*$ by $\tilde{\mathcal{E}}_{x^*,t}$, that is

$$\tilde{\mathcal{E}}_{x^*,t} \equiv E_t\left[\tilde{x}_{t+1}^*\right],$$

and assume the following dynamic for $\tilde{\mathcal{E}}_{x^*,t}$,

$$\tilde{\mathcal{E}}_{x^*,t+1} = \mu^* + \phi^* \tilde{\mathcal{E}}_{x^*,t} + \psi^* \varepsilon_{z,t+1}.$$

We now focus on the derivation of $\mathcal{E}_{xcr,t}$, we have

$$\begin{aligned}
\mathcal{E}_{xcr,t} &= \Delta_c i_t + E_t[\Delta s_{t+1}] \\
&= \delta_0^* + \tau_t^* + \delta_{\zeta^*}^\top \zeta_t^* + E_t[\Delta \bar{s}_{t+1}] + E_t[\Delta \tilde{s}_{t+1}] \\
&= \delta_0^* + \tau_t^* + \tilde{\lambda}_{\zeta^*}^\top \left(I - \phi_*^{\mathbb{Q}}\right) \zeta_t^* + \tilde{\lambda}_{\zeta^*}^\top E_t[\Delta \zeta_{t+1}^*] \\
&= -\tilde{\lambda}_{\zeta^*}^\top \mu_*^{\mathbb{Q}} + \frac{1}{2}\tilde{\lambda}_{\zeta^*}^\top \Omega_{\zeta^*} \tilde{\lambda}_{\zeta^*} + \bar{\tau}_t^* + \tilde{\tau}_t^* - \tilde{\lambda}_{\zeta^*}^\top \phi_*^{\mathbb{Q}} \zeta_t^* + \tilde{\lambda}_{\zeta^*}^\top E_t[\zeta_{t+1}^*] \\
&= \bar{\tau}_t^* + \tilde{\tau}_t^* + \tilde{\lambda}_{\zeta^*}^\top \left[E_t[\zeta_{t+1}^*] - \left(\mu_*^{\mathbb{Q}} + \phi_*^{\mathbb{Q}} \zeta_t^*\right)\right] + \frac{1}{2}\tilde{\lambda}_{\zeta^*}^\top \Omega_{\zeta^*} \tilde{\lambda}_{\zeta^*} \\
&= \bar{\tau}_t^* + \tilde{\tau}_t^* + \tilde{\lambda}_{\zeta^*}^\top \left[\tilde{\mathcal{E}}_{\zeta^*,t} - \left(\mu_*^{\mathbb{Q}} + \phi_*^{\mathbb{Q}} \zeta_t^*\right)\right] + \frac{1}{2}\tilde{\lambda}_{\zeta^*}^\top \Omega_{\zeta^*} \tilde{\lambda}_{\zeta^*}.
\end{aligned}$$

We estimate physical parameters $\bar{\sigma}^*$, μ^*, ϕ^* and ψ^* by maximizing the Gaussian likelihood for $\varepsilon_{z,t}$, assuming $\varepsilon_{z,t} \sim N\left(0, \frac{1}{T}\sum_{t=1}^{T} \varepsilon_{z,t}\varepsilon_{z,t}^\top\right)$.

AIII Stochastic Discount Factors

Domestic SDF The specification of the domestic bond market implies that the domestic stochastic discount factor has the following form:

$$\begin{aligned} M_{t+1} &= e^{-i_t} M_{\tau,t+1} M_{\zeta,t+1} M_{\eta,t+1} \\ &= e^{-i_t} \exp\left(\lambda_{\tau,t}\left(\tau_{t+1} - \mathcal{E}_{\tau,t}\right) + \lambda_{\zeta,t}\left(\zeta_{t+1} - \mathcal{E}_{\zeta,t}\right) + \lambda_{\eta,t}\left(\eta_{t+1} - \mathcal{E}_{\eta,t}\right) + \mathcal{J}_t\right), \end{aligned} \tag{1}$$

where only the first two components are relevant to price zero-coupon bonds, the third component will be determined below, $E_t[M_{t+1}] = \exp(-i_t)$ determines the Jensen term $\mathcal{J}_t$, and the prices of risk are given by[14]:

$$\lambda_{\zeta,t} = \Omega_{\zeta}^{-1}\left(E_t^{\mathbb{Q}}[\Delta\zeta_{t+1}] - E_t[\Delta\zeta_{t+1}]\right) \tag{2}$$

$$\lambda_{\tau,t} = \Omega_{\tau}^{-1}\left(E_t^{\mathbb{Q}}[\Delta\tau_{t+1}] - E_t[\Delta\tau_{t+1}]\right) = -\Omega_{\tau}^{-1} E_t[\Delta\tau_{t+1}]. \tag{3}$$

We also include in the SDF the vector of N_η gaussian innovations $\varepsilon_{\eta,t+1} = \left(\eta_{t+1} - \mathcal{E}_{\eta,t}\right)$ with covariance matrix $\Omega_{\eta,t}$ to the SDF. These innovations will be relevant for the exchange rate dynamics but they are uncorrelated with the yield factors innovations $\varepsilon_{x,t+1}$, and they are not spanned by bond yields, following Chernov and Creal (2023).

Foreign SDF The specification of the foreign bond market implies that the foreign SDF has the following form:

$$\begin{aligned} M^*_{t+1} =& e^{-i_t} M_{\tau,t+1} M_{\zeta,t+1} \times \\ & e^{-\Delta_c i_t} \exp\left(\lambda^*_{\tau^*,t}\left(\tau^*_{t+1} - \mathcal{E}_{\tau^*,t}\right) + \lambda^*_{\zeta^*,t}\left(\zeta^*_{t+1} - \mathcal{E}_{\zeta^*,t}\right) + \lambda^*_{\eta,t}\left(\eta_{t+1} - E_t[\eta_{t+1} | \zeta^*_{t+1}, \tau^*_{t+1}]\right) + \mathcal{J}^*_t\right), \end{aligned}$$

where $E_t[M^*_{t+1}] = \exp(-i^*_t)$ determines the Jensen term $\mathcal{J}^*_t$ and the prices of risk are given by:

$$\begin{aligned} \lambda^*_{\zeta^*,t} &= \Omega_{\zeta^*}^{-1}\left(E_t^{\mathbb{Q}}[\Delta\zeta^*_{t+1}] - E_t[\Delta\zeta^*_{t+1}]\right) \\ \lambda_{\tau^*,t} &= -\Omega_{\tau^*}^{-1} E_t[\Delta\tau^*_{t+1}]. \end{aligned} \tag{4}$$

We allow for correlations matrix between η_{t+1} and any element of x^*_{t+1}, so that the last component of the foreign SDF is the projection of η_{t+1} innovations on x^*_{t+1}. The domestic and foreign SDFs share the innovations due to η_{t+1} with different prices of risk and, as we show in the next section, this term will contribute to the exchange rate variations through the permanent component of the SDFs.[15]

Under the asset market view of the exchange rate, $\Delta s_{t+1} = \Delta_c \ln M_{t+1}$, the exchange

[14] The last equality follows from the fact that $E_t^{\mathbb{Q}}[\Delta\tau_{t+1}] = 0$.

[15] That is, $\eta^{\mathbb{P}}_{t+1}$ only causes permanent changes to the exchange rate.

rate dynamics are given by:

$$
\begin{aligned}
xcr_{t+1} = & \left(\lambda_{x^*,t} - \Omega_{x^*}^{-1}\Omega_{\eta x^*}^{\top}\lambda_{\eta,t}^*\right)^{\top}\left(x_{t+1}^* - \mathcal{E}_{x^*,t}\right) + \left(\lambda_{\eta,t}^* - \lambda_{\eta,t}\right)^{\top}\left(\eta_{t+1} - \mathcal{E}_{\eta,t}\right) \\
& - \frac{1}{2}\lambda_{\tau*,t}^{*\top}\Omega_{\tau^*}\lambda_{\tau*,t}^* - \frac{1}{2}\lambda_{\zeta^*,t}^{*\top}\Omega_{\zeta^*}\lambda_{\zeta^*,t}^* - \frac{1}{2}\lambda_{\eta,t}^{*\top}\bar{\Omega}_{\eta}\lambda_{\eta,t}^* + \frac{1}{2}\lambda_{\eta,t}^{\top}\Omega_{\eta,t}\lambda_{\eta,t},
\end{aligned} \tag{5}
$$

where $\bar{\Omega}_{\eta} = \Omega_{\eta} - \Omega_{\eta x^*}\Omega_{x^*}^{-1}\Omega_{\eta x^*}^{\top}$ and the prices of risk $(\lambda_{\eta,t}, \lambda_{\eta,t}^*)$ are not yet defined. Putting these results together, we get the following results.

$$
h_t^b \equiv \lambda_{x^*,t}^{\top}\Omega_{x^*}\lambda_{x^*,t}
$$

Proposition 3 *Define the currency risk premium $crp_t = E_t[xcr_{t+1}]$ as follows:*

$$
crp_t = -\frac{1}{2}\lambda_{\tau*,t}^{*\top}\Omega_{\tau^*}\lambda_{\tau*,t}^* - \frac{1}{2}\lambda_{\zeta^*,t}^{*\top}\Omega_{\zeta^*}\lambda_{\zeta^*,t}^* - \frac{1}{2}\lambda_{\eta,t}^{*\top}\bar{\Omega}_{\eta}\lambda_{\eta,t}^* + \frac{1}{2}\lambda_{\eta,t}^{\top}\Omega_{\eta}\lambda_{\eta,t} \tag{6}
$$

and the conditional variance $h_t = var_t[xcr_{t+1}]$ by:

$$
h_t = (\Delta_c\lambda_{\eta,t})^{\top}\bar{\Omega}_{\eta}(\Delta_c\lambda_{\eta,t}) + \lambda_{\eta_t}^{\top}\Omega_{\eta x^*}\Omega_{x^*}^{-1}\Omega_{\eta x^*}^{\top}\lambda_{\eta,t} + \lambda_{x^*,t}^{\top}\Omega_{x^*}\lambda_{x^*,t} - 2\lambda_{\eta,t}^{\top}\Omega_{\eta x^*}\lambda_{x^*,t}. \tag{7}
$$

Given Equations (35)-(36)*, the prices of risk $(\lambda_{\eta,t}, \lambda_{\eta,t}^*)$ solve the following non-linear equations:*

$$
\begin{aligned}
\lambda_{\eta^*,t} = & \frac{\Omega_{\eta}\lambda_{\eta,t}^2 - \left(\Omega_{\eta x^*}\lambda_{x^*,t}\right)\lambda_{\eta,t} - \left(\pi_t + \frac{h_t}{2}\right)}{\bar{\Omega}_{\eta}\lambda_{\eta,t}} \\
\lambda_{\eta_t} = & \frac{\bar{\Omega}_{\eta}\lambda_{\eta^*,t}^2 - \left(\frac{h_t}{2} - \lambda_{x^*,t}^{\top}\Omega_{x^*}\lambda_{x^*,t} - \pi_t\right)}{\bar{\Omega}_{\eta}\lambda_{\eta^*,t} + \Omega_{\eta x^*}\lambda_{x^*,t}}
\end{aligned}
$$

AIII.1 Data and Additional Results

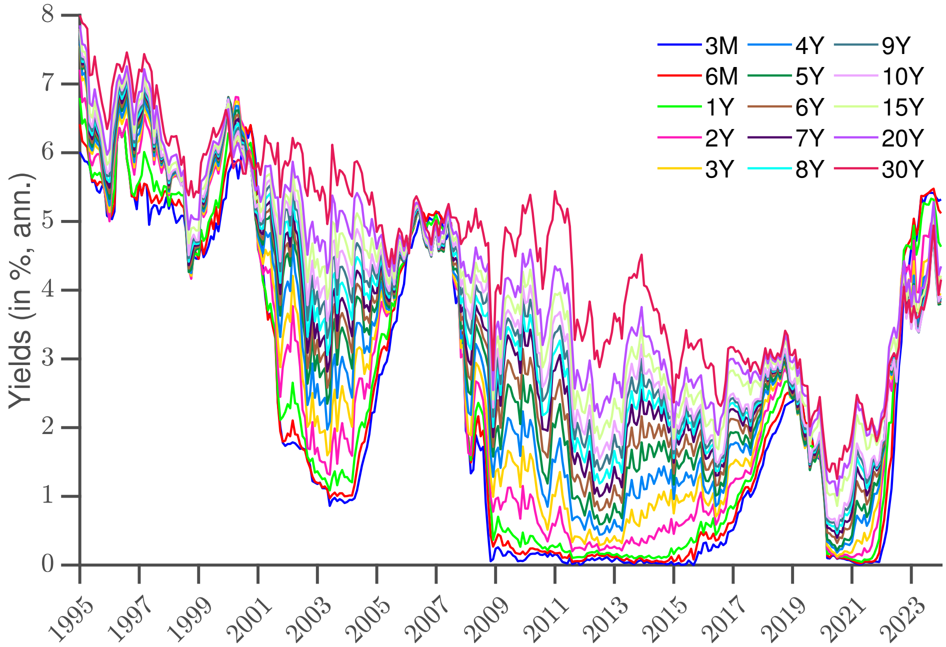

Figure A-1: United States: Annualized Yields, various maturities
The figure shows the time series dynamics of yields for the United States. Values are expressed in annualized percentage. The sample period is January 1995 to January 2024.

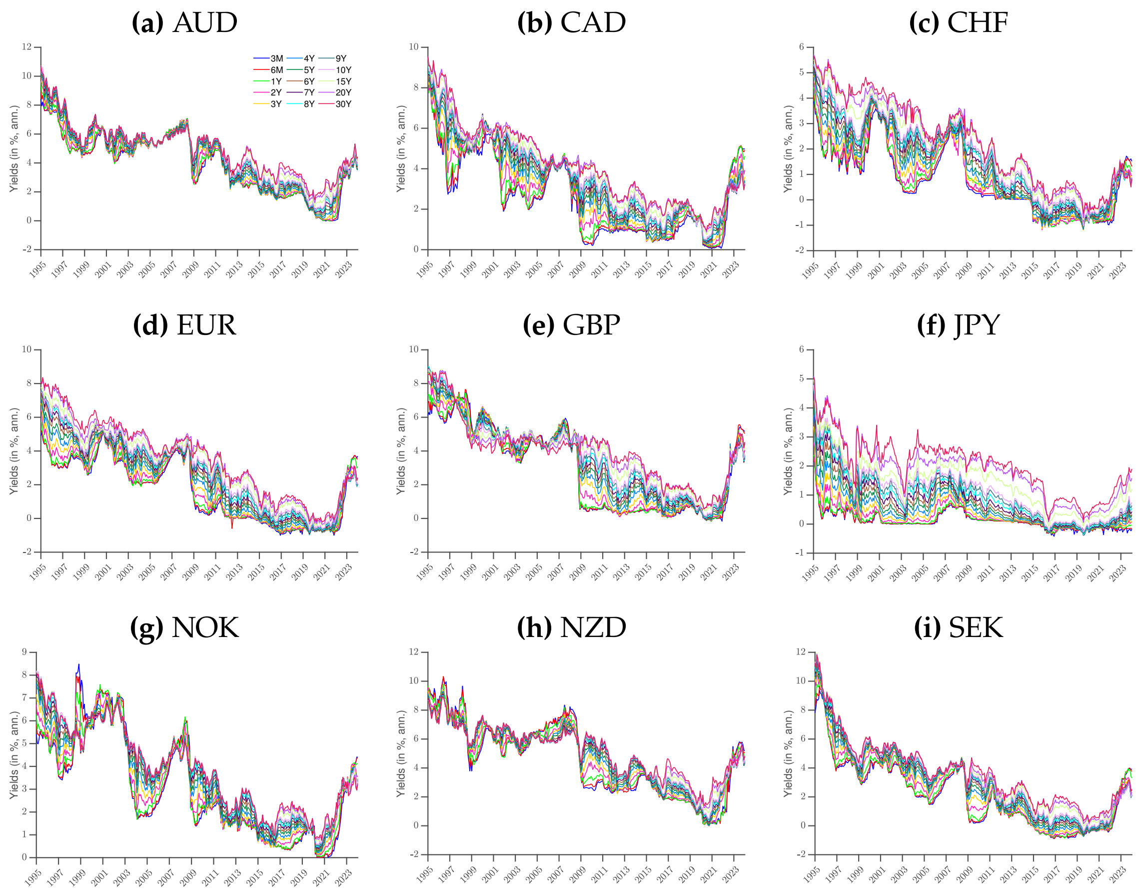

Figure A-2: Small Open Economies: Annualized Yields, various maturities
The figure shows the time series dynamics of yields for each small open economy. Values are expressed in annualized percentage. The sample period is January 1995 to January 2024.

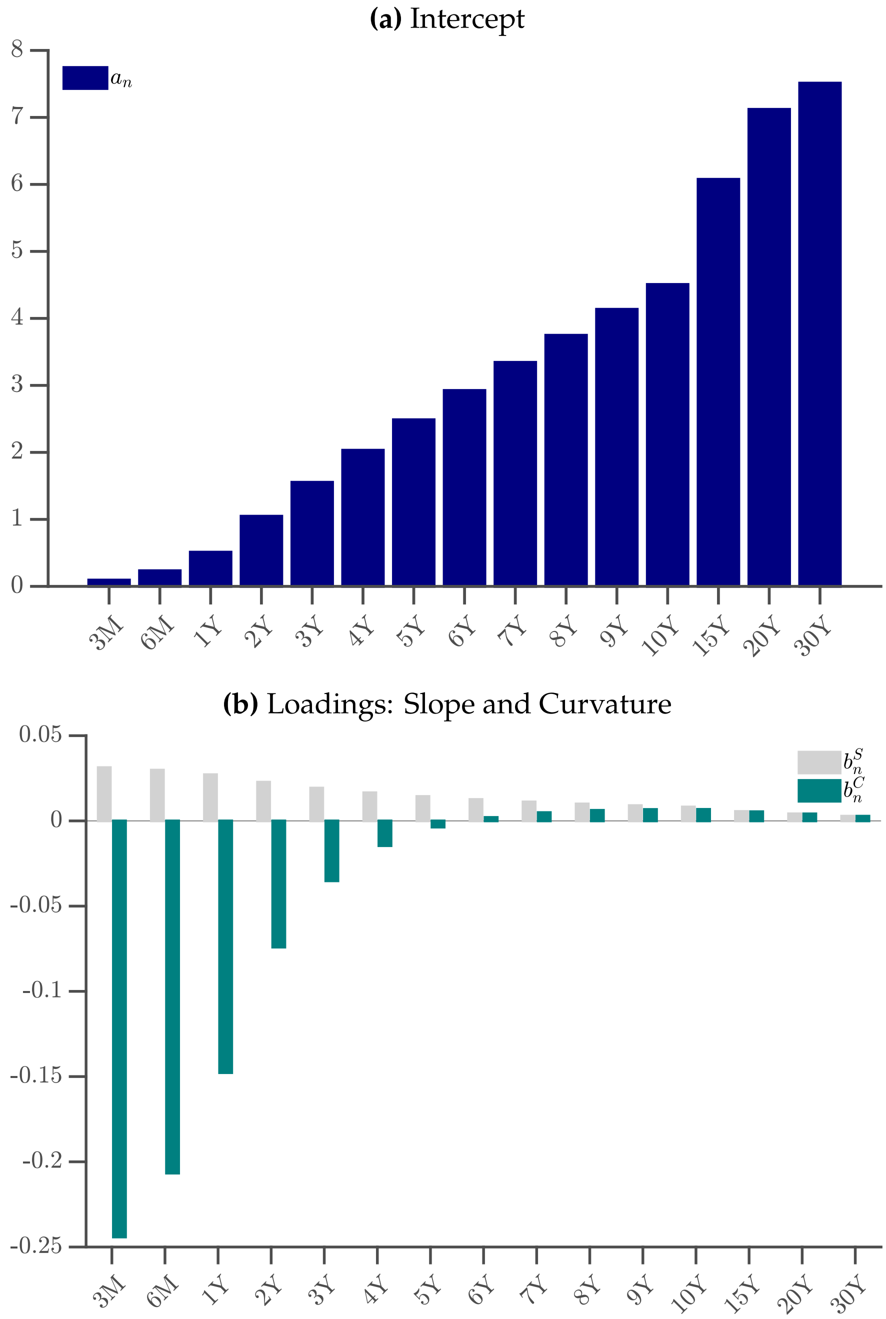

Figure A-3: Factor Loadings: United States

The figure shows the estimated factor loadings. Loadings on the levels are not presented, as they are set to 1. The sample period is January 1995 to January 2024.

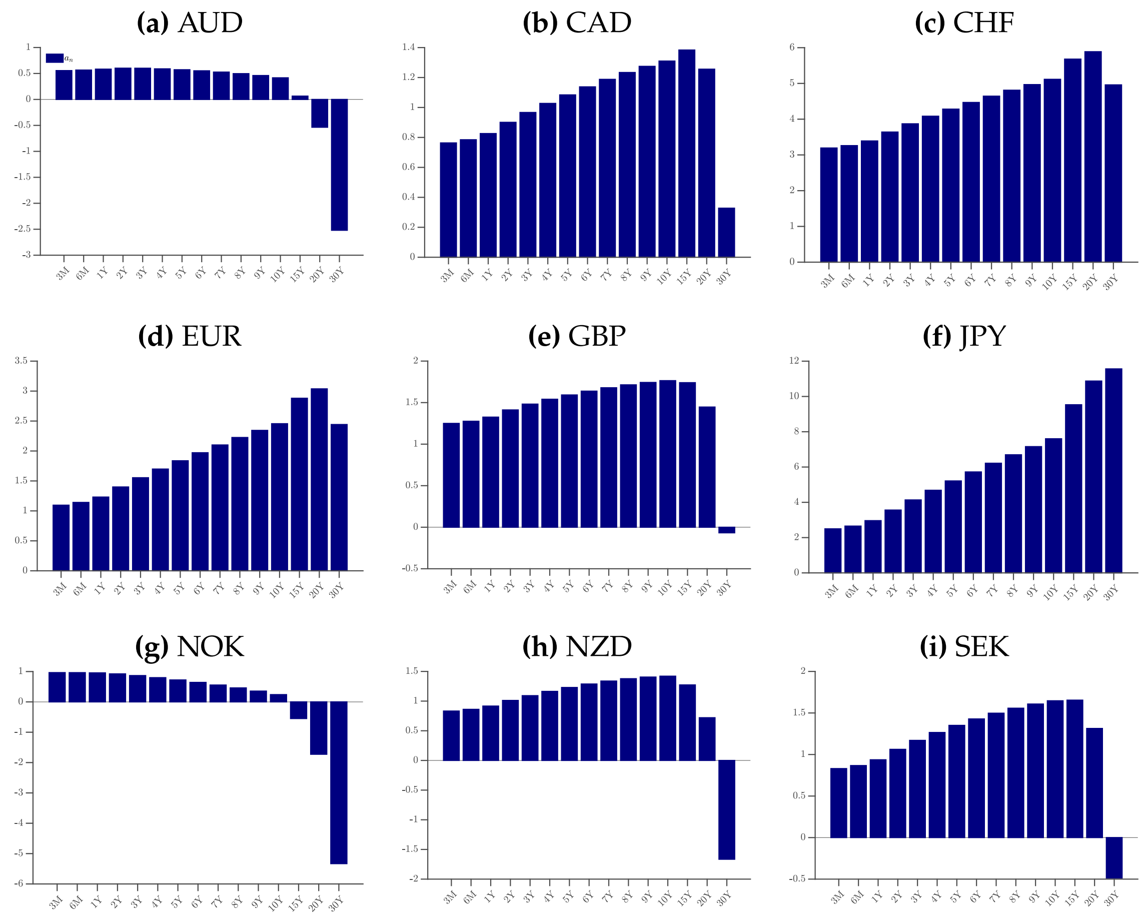

Figure A-4: Factor Loadings - Intercept: Small Open Economies
The figure shows the estimated factor loadings of the intercept for every small open economies. The sample period is January 1995 to January 2024.

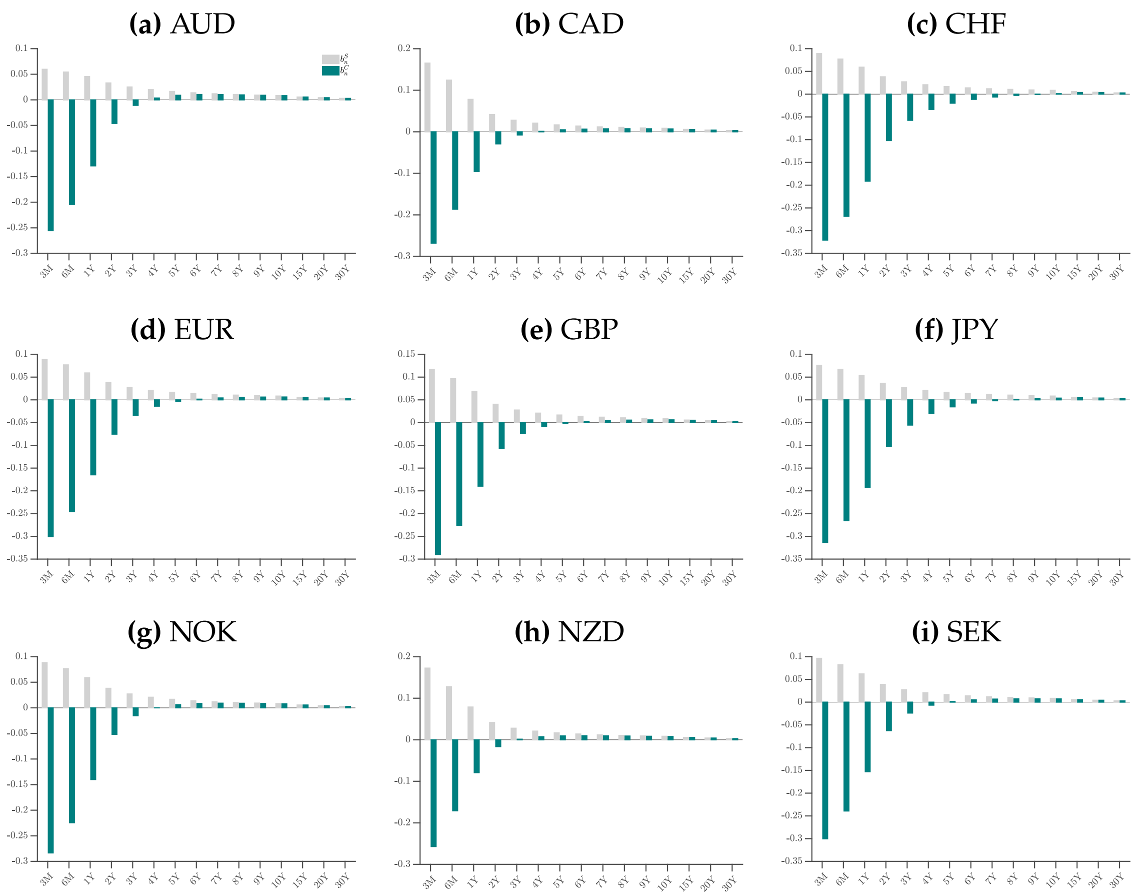

Figure A-5: Factor Loadings - Slope and Curvature: Small Open Economies
The figure shows the estimated factor loadings of the slope and curvature for every small open economies. The sample period is January 1995 to January 2024.

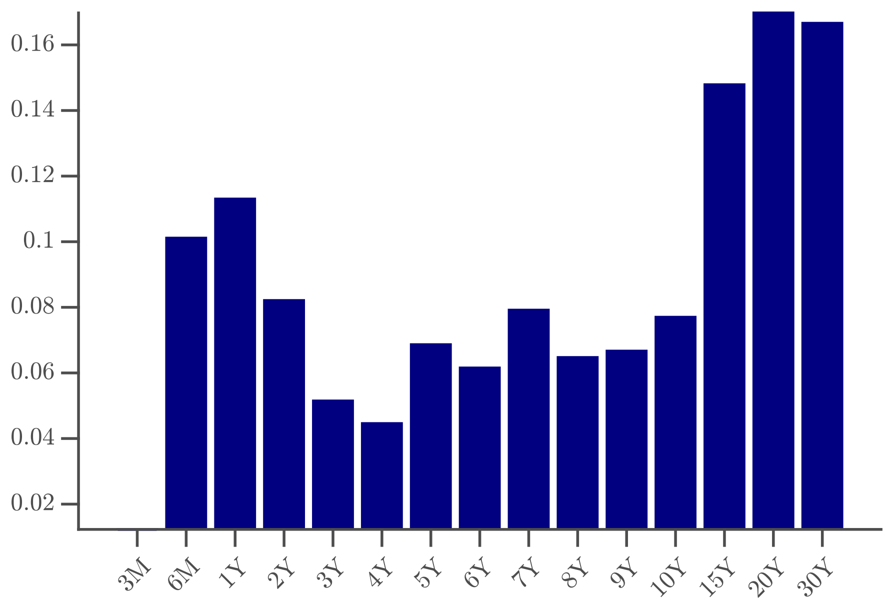

Figure A-6: Root Mean Square Error: United States
The figure shows root mean square error for different maturities. The sample period is January 1995 to January 2024.

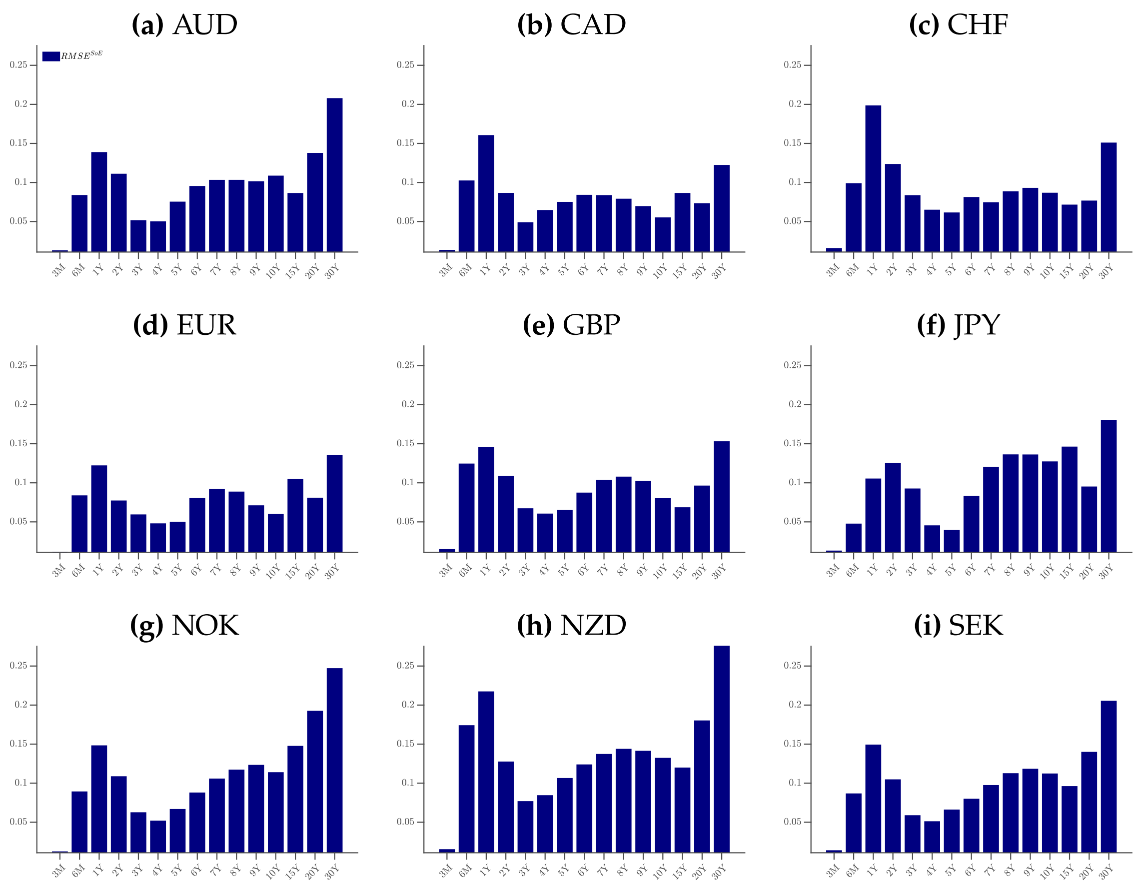

Figure A-7: Root Mean Square Error: Small Open Economies
The figure shows root mean square error for different maturities for each small open economy. The sample period is January 1995 to January 2024.